

## Examples of bounds on Frobenius number for ESM lattices

Erdős, Graham (1972):

$$\mathcal{F} \leq 2a_N \left\lfloor \frac{a_1}{N} \right\rfloor - a_1. \quad (1)$$

Vitek (1975):

$$\mathcal{F} \leq \left\lfloor \frac{(a_2 - 1)(a_N - 2)}{2} \right\rfloor - 1. \quad (2)$$

Selmer (1977):

$$\mathcal{F} \leq 2a_{N-1} \left\lfloor \frac{a_N}{N} \right\rfloor - a_N. \quad (3)$$

Beck, Diaz, Robins (2002):

$$\mathcal{F} \leq \frac{1}{2} \left( \sqrt{a_1 a_2 a_3 (a_1 + a_2 + a_3)} - a_1 - a_2 - a_3 \right). \quad (4)$$

Fukshansky, Robins (2005, ESM case):

$$\mathcal{F} \leq \left\lfloor \frac{(N-1)^2 \sum_{i=1}^N a_i \sqrt{\|\mathbf{a}\|^2 - a_i^2}}{(\|\mathbf{a}\|^{N-2} \omega_{N-1})^{\frac{1}{N-1}}} + 1 \right\rfloor. \quad (5)$$

$N = 4$ :

<i>4-tuple</i> $\mathbf{a}$	$\lambda_{\mathbf{a}}$	<i>min</i> (1) - (4)	(5)
9337, 9961, 11593, 67367	$\sqrt{1802}$	91235853 (4)	10995433
33199, 38351, 47759, 152057	$\sqrt{3218}$	1346684400 (4)	55055950

$N = 5$ :

<i>5-tuple</i> $\mathbf{a}$	$\lambda_{\mathbf{a}}$	<i>min</i> (1) - (4)	(5)
39221, 46967, 47869, 62839, 206749	$\sqrt{524}$	1719019240 (4)	66231577
1867558, 2348176, 2918749, 5249843, 26695349	$\sqrt{5591}$	4778060891200 (4)	14595157176

$N = 6$ :

<i>6-tuple</i> $\mathbf{a}$	$\lambda_{\mathbf{a}}$	<i>min</i> (1) - (4)	(5)
6595, 90709, 110483, 121833, 147472, 462217	$\sqrt{209}$	1015946371 (1)	168600688
5958323, 14864655, 19945128, 28191201, 28507523, 117697394	$\sqrt{1915}$	134180083643479 (4)	104669816535