Placement test instructions:

There are three exams included in this packet, one for placement in each semester of our calculus sequence, and solutions are provided.

Test I checks readiness for Math 30, first semester calculus. It is multiple choice and getting 13 out of 25 or more right probably indicates readiness for Math 30 (accounting for summer lag etc.). The test should take a bit more than an hour, but the timing is not so important. What is important is that the student should be familiar with the questions and have the general feeling that she knows how to solve them, or that she could get back up to speed on the necessary topics with ease. In this case, we encourage you to try Math 30 and to seek our help in reviewing the exam problems.

Test II checks readiness for Math 31, second semester calculus. The exact same comments and advice apply as for test I. In both these exams, it may be the case that simply working out the problems with the help of an old book will be enough to get you back up to speed for the course you have in mind. Feel free to come ask math faculty for help on the questions if you want to treat the exams as refresher exercises.

Test III checks readiness for Math 32, third semester calculus. It is not multiple choice and there are only 7 questions. A good showing on at least 4 of the seven questions probably indicates that you are ready for Math 32, but once again, if you have the general feeling that you could do the questions after a bit of review, you are encouraged to try Math 32 and to use the exam as a review exercise.

Placement Test 1
To check your preparation for Math 30 (Calculus I)

your name (please print) _____

email_____

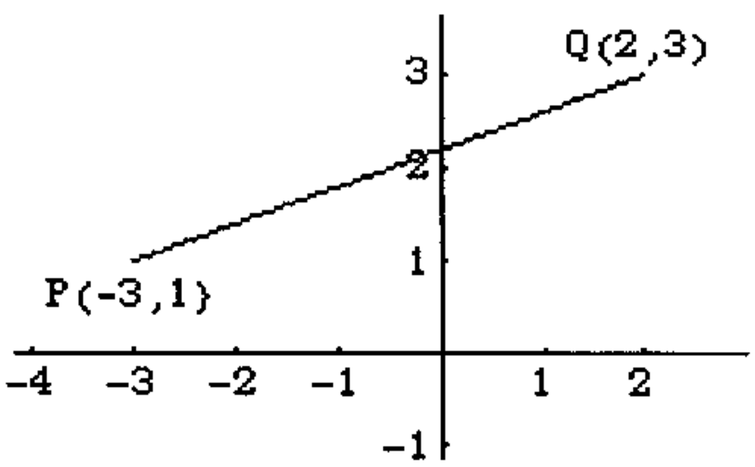
1. Let $f(x)=\dfrac{x-1}{\sqrt{x+2}}$. Then the domain of the function is

a) the half line $[0,+\infty)$ _____
b) the interval $(-2,0]$ _____
c) the half line $(-2,+\infty)$ _____
d) the entire real line _____
e) the half line $[-2,+\infty)$ _____

2. Let $f(x)=2x+3$ and $g(x)=1-x$. Then $g(f(x))=$

a) $2x+1$ _____
b) $-2x+1$ _____
c) $2x+2$ _____
d) $-(2x+2)$ _____
e) $-(2x-1)$ _____

3. The distance between P and Q is



a) 7 _____
b) -7 _____
c) $\sqrt{29}$ _____
d) 3 _____
e) $\sqrt{5}$ _____

4. $\sqrt[5]{64x^6y^8z^{10}}\ =$

a) $2xyz^2\sqrt[5]{xy^3}$ _____
b) $2xyz^2\sqrt[5]{2x^2y^3}$ _____
c) $2xyz^2\sqrt[5]{2xy^3}$ _____
d) $4xyz^2\sqrt[5]{2xy^3}$ _____
e) none of the above _____

5. If $x \neq 1$ then $(x^3 - 1)/(x-1)$ is equal to

a) $x^2 - 1$ _____
b) $x^2 + x + 1$ _____
c) $x^2 - x + 1$ _____
d) $x^2 + 1$ _____
e) none of the above _____

6. The equation $x + 2y = 1$ represents a line L of the xy-plane.
The equation of the line M parallel to L and containing the point (–1,2) is

a) $x - 2y = -5$ _____
b) $x + 2y = 3$ _____
c) $x + 2y = 2$ _____
d) $-x + 2y = 3$ _____
e) none of the above _____

7. The equation $x + 2y = 1$ represents a line L of the xy-plane.
The equation of the line N perpendicular to L and containing the point (1,–2) is

a) $2x - y = 4$ _____
b) $2x + y = 0$ _____
c) $x + y = -1$ _____
d) $-2x + 3y = -8$ _____
e) none of the above _____

8. Which of the following are factors of $x^4 - 81$

I. $x + 3$ II. $x - 3$ III. $x^2 + 9$

a) I only _____
b) II only _____
c) III only _____
d) I and II _____
e) All three _____

9. Let $f(x) = x^2 + 1$. Then f(x+h)=

a) $x^2 + 2xh + h^2 + 1$ _____
b) $x^2 + h^2 + 1$ _____
c) $(x + h + 1)^2$ _____
d) $x^2 + h + 1$ _____
e) none of the above _____

10. The values of a such that the solutions of the quadratic equation $x^2 - ax + 4 = 0$ are real and distinct are

a) all values _____
b) $a < -4$ or $a > 4$ _____
c) $a < 0$ _____
d) $a > 0$ _____
e) none of the above _____

11. If x<y then

a) $1/x > 1/y$ _____
b) $x^2 < y^2$ _____
c) $xy < y^2$ _____
d) $2x < 2y$ _____
e) $|x| < |y|$ _____

12. Let $f(x) = \dfrac{x-1}{\sqrt{x+2}}$. Then the range of the function is

a) the half line $[-3,+\infty)$ _____
b) the half line $(-3,+\infty)$ _____
c) the entire real line _____
d) the half line $(-\infty,0]$ _____
e) none of the above _____

13. All values of x such that $|x-2|< 5$ are

a) $2 < x < 5$ _____
b) $-3 < x < 7$ _____
c) $-5 < x < 3$ _____
d) $|x| < 3$ _____
e) $3 < x < 7$ _____

14. The center of the circle $x^2 + y^2 - 4x + 6y = -9$ is

a) (2,3) _____
b) (–2,3) _____
c) (2,–3) _____
d) (–2,–3) _____
e) none of the above _____

15. The equation $y = x^2 - 4x + 3$ represents

a) a parabola _____
b) one line _____
c) 2 lines _____
d) a circle _____
e) a hyperbola _____

16. You are given the graph of a function f(x). You want to get a function g(x) such that its graph is obtained from the graph of f by rigidly moving it 3 units to the right and 2 units down. Then

a) g(x)=f(x–3)+2 _____
b) g(x)=f(x+3)+2 _____
c) g(x)=f(x–3)–2 _____
d) g(x)=f(x+3)–2 _____
e) none of the above _____

17. The line $y = 3x - 1$ intersects the parabola $2y = x^2 - 2$ exactly at

a) (0,–1), (6,17) _____
b) (–1,0), (6,17) _____
c) (1,2), (0,–2) _____
d) (1,–0.5), (–1,–4) _____
e) (2,5), (2,1) _____

18. The two most common temperature scales are Celsius and Fahrenheit. You are informed that 32 Fahrenheit corresponds to 0 Celsius and 212 Fahrenheit correspond to 100 Celsius. Then, assuming that the graph of the relation is a straight line, you derive

a) C=0.5F–32 _____
b) C=5(F–32)/9 _____
c) C=5F/9–-32 _____
d) C=0.4F– 32 _____
e) F=2C+32 _____

19. While shopping for a new dress you found one you like. Too expensive! After a month the price had been discounted by 30%. To entice you to buy it you are offered an additional discount of 10% on the new price. What is the discount you receive over the price you found the first time?

a) 40% _____
b) 35% _____
c) depends on the price _____
d) 45% _____
e) none of the above _____

20. A car agency charges $ 35 per day for renting a certain kind of car, plus 55 cents per mile. Gasoline costs $ 2.07 per gallon and the car gets 30 miles per gallon.
Express the total cost C (in dollars) of driving the car on a 3-day trip covering x miles as a function of x.

a) $105 + 0.55x + 62.1/x$ _____
b) $105 + 2.62x/30$ _____
c) $105 + 0.55/x + 2.07x/30$ _____
d) $105 + 0.619x$ _____
e) $106.55 + 0.69x$ _____

21. A function f(x) is periodic of period 2. Choose the correct statement.

a) $f(2x) = f(x)$ for every x. _____

b) $f(x+3) = f(x)$ for every x. _____

c) $f(x+2) = f(x)$ for every x. _____

d) $f(x-2) = f(-x)$ for every x. _____

e) none of the above _____

22. Suppose that the measure α (in radians) of an angle A is between $\pi/2$ and π. If $\sin\alpha = 4/7$ then cosα=

a) 3/7 _____

b) 4/3 _____

c)–4/3 _____

d) $\sqrt{33}/7$ _____

e)–$\sqrt{33}/7$ _____

23. If $\log_5(x-7) = 2$, then what is the value of x?

a) 18 _____

b) 39 _____

c) 25 _____

d) 32 _____

e) 17 _____

24. Suppose that $10^{3t-2} = 5$. Then t=

a) 0 _____

b) 1/3 _____

c) $(2-\log_{10}5)/3$ _____

d) $(2+\log_{10}5)/3$ _____

e) none of the above _____

25. $\log_2 a - \log_2(a-3) =$

a) $\log_2(a/(a-3))$ _____

b) $\log_2(2a-3)$ _____

c) $a^2 - 3a$ _____

d) $\log_2(a^2-3a)$ _____

e) $\log_2(a^2+3a)$ _____

Solutions to Placement Test I.

1.c
2.d
3.c
4.c
5.b
6.b
7.a
8.e
9.a
10.b
11.d
12.c
13.b
14.c
15.a
16.c
17.a
18.b
19.e
20.d
21.c
22.e
23.d
24.d
25.a

Placement Test Calculus II

To qualify for Math 31

Your name_____

The last four digits of your student ID_____

1. $\lim_{x\to 1}\dfrac{x^3-1}{x-1}=$

a) 0 ____
b) 1 ____
c) does not exist ____
d) 3 ____
e) 2 ____

2. $\lim_{x\to\infty}\dfrac{\sqrt{2x^2+x+1}}{x}=$

a) 0 ____
b) 1 ____
c) $\sqrt{2}$ ____
d) –1 ____
e) 1.4145 ____

3. Let $f(x)=x^3-2x$. All values of x such that the tangent line to the graph of f(x) is horizontal are

a) 0 and 1 ____
b) $\pm 2/\sqrt{3}$ ____
c) $\pm\sqrt{2/3}$ ____
d) 0 and 2 ____
e) none of the above ____

4. Let $f(x)$ and $g(x)$ be differentiable and such that $f'(x)=g'(x)$ for all x. Then

a) f(x)=g(x) ____
b) f(x)g(x)=constant ____
c) f(x)/g(x)=constant ____
d) f(x)–g(x)=constant ____
e) f(x)+g(x)=constant ____

5. The critical values of $f(x)=x-\dfrac{\cos \pi x}{\pi}$ for $0\le x\le 2$ are

a) 0 and 1 ____
b) 1.5 ____
c) 0.25 and 0.75 ____
d) 1 and 1.5 ____

e) none of the above ____

6. The linear approximation of $f(x) = \sqrt{x+3}$ at x=1 is

a) x+3 ____
b) (7+x)/4 ____
c) (5+x)/4 ____
d) x+2 ____
e) x+1 ____

7. The maximum value of $f(x)$ is 2 and $f'(x) = 2 - 2x$. Then $f(x)$=

a) $-x^2 + 2x + 1$ ____
b) $-x^2 + 2x + 2$ ____
c) $-x^2 + 2x$ ____
d) $-x^2 + 2x - 2$ ____
e) $-x^2 + 2x - 1$ ____

8. Let $f(x) = x^3/3 + x^2/2 + 6x + 8$. Then the minimum value of this function in the closed interval [0,3] is

a) 2/3 ____
b) 3/2 ____
c) 2 ____
d) –2 ____
e) 8 ____

9. A rectangular enclosure of 20000 ft^2 has one side along a road. Fencing this side costs \$3/foot. Fencing the remaining 3 sides cost \$1/foot. The minimal cost of doing this job is

a) \$200 ____
b) \$800 ____
c) \$950 ____
d) \$300 ____
e) \$1250 ____

10. Let

$$f(x) = \begin{cases} x^3 & x \le 2 \\ x^2 + kx & 2 < x \end{cases}$$

For what value of k will f be continuous at x=2?

a) 32 ____
b) 14 ____
c) 7 ____
d) 5 ____
e) 2 ____

11. Let $x^2y+\sqrt{y}+x=1$. Notice that the curve passes through the point (0,1). The tangent line at that point is

a) $x-2y=-2$ ____
b) $x-y=-1$ ____
c) $x=0$ ____
d) $2x+y=1$ ____
e) $2x+y=2$ ____

12. Let $G(x)=\int^{x} g(s)ds$. Then $G'(x)=$

a) $g'(x)-g'(1)$ ____
b) $g(x)-g(1)$ ____
c) $g(x)$ ____
d) $G(x)-G(1)$ ____
e) $G(x)$ ____

13. Let $h(x)=f(g(x))$, $f'(4)=5$, $g'(3)=8$, $g(3)=4$. Then $h'(3)=$

a) 40 ____
b) 20 ____
c) we do not know ____
d) 8 ____
e) 10 ____

14. Assume that $f''(x)f'(x)<0$ in [0,2]. Among the possibilities listed below select the one which could be true.

a) f is increasing in [0,2] and concave up. ____
b) f is increasing in [0,2] and concave down. ____
c) f is decreasing in [0,2] and concave down. ____
d) f is constant in [0,2] ____
e) f has a maximum value in 0 and in 2. ____

15. Assume that $f'(c)=0$, $f'(x)<0$ when $x<c$ and $f'(x)>0$ when $x>c$. Then

a) f has an absolute maximum at c ____
b) f has an inflection point at c ____
c) f is decreasing at c ____
d) f is increasing at c ____
e) f has an absolute minimum at c ____

16. For what values of c in (–1,2) is the tangent line to the graph of $f(x)=x^3-x$ at the point $(c, f(c))$ parallel to the line joining $(-1, f(-1))$ with $(2, f(2))$?

a) 2 ____
b) 0 ____
c) 1 and –1 ____

d) 1 ____
e) none ____

17. Gas is escaping from a spherical balloon at a rate of $2\,ft^3/\min$. How fast is the surface area decreasing when $r = 12\,ft$?

Recall that the volume and surface area of a sphere are $\frac{4\pi r^3}{3}$ and $4\pi r^2$ respectively.

a) $2/3\,ft^2/\min$ ____
b) $1/3\,ft^2/\min$ ____
c) $2\,ft^2/\min$ ____
d) $1/4\,ft^2/\min$ ____
e) $0.2\,ft^2/\min$ ____

18. Let $s(t) = t^3 - 6t^2 + 9t + 4$, denote the position of a particle at time t. When the acceleration is 0 the velocity is

a) -3 ____
b) 2 ____
c) 3 ____
d) 1 ____
e) 6 ____

19. Suppose that f is continuous at 2, $f(2) > 0$, and f is defined in $(-1,3)$. Choose the correct statement.

a) there exists an open interval around 2 where f is positive ____
b) f is increasing at 2 ____
c) f is decreasing at 2 ____
d) f is concave up in an open interval around 2 ____
e) none of the above ____

20. Let $f(x) = \arctan(x)$ and $g(x) = \sin(x)$. Let $h(x) = f(g(x))$. Then $h'(x) =$

a) $\frac{\cos(\arctan x)}{1+x^2}$ ____
b) $\frac{1}{\cos x}$ ____
c) $\frac{\cos x}{1+\sin^2 x}$ ____
d) $(1+x^2)^{-3/2}$ ____
e) $\frac{\sin x}{1+\cos^2 x}$ ____

21. $\int_0^3 \sqrt{9-x^2}\,dx$ represents the area of

a) a quarte circle of radius 3 ____
b) a quarter circle of radius 5 ____
c) half circle of radius 3 ____

d) half circle of radius 5 ____
e) half circle of radius 9 ____

22. Let $f(x)=\begin{cases}-1 & \text{for} \quad 0\le x\le 1\\ x & \text{for} \quad 1<x\le 2\end{cases}$. Then $\int_0^2 f(x)dx =$

a) 1 ____
b) 2 ____
c) 1/2 ____
d) –1/2 ____
e) 0 ____

23. Assume that f and g are continuous and $\int_1^2 f(x)dx = \int_1^2 g(x)dx$. Then in [1,2] , which one of the following must hold?

a) $f(x)=g(x)$ ____
b) $f(x)>g(x)$ ____
c) $f(x)=g(x)=0$ ____
d) $f(x)-g(x)$ changes sign in [1,2] ____
e) none of the above ____

24. The area of the bounded region enclosed by the x-axis and the graph of the function $f(x)=x((2x-3)^2$ is

a) $27/4$ ____
b) $27/16$ ____
c) $9/4$ ____
d) $81/4$ ____
e) 3 ____

25. Let $f(x)=\int_x^1 \sin(y^2+y)dy$. Then $f'(x)$ is equal to

a) 0 ____
b) does not exist ____
c) $\sin(x^2+x)$ ____
d) $\sin(x^2-x)$ ____
e) $-\sin(x^2+x)$ ____

Solutions to placement test II

1.d
2.c
3.c
4.d
5.b
6.b
7.a
8.e
9.b
10e
11.d
12.c
13.a
14.b
15.e
16.d
17.b
18.a
19.a
20.c
21.a
22.c
23.e
24.b
25.e

Placement Test Calculus III
To qualify for Math 32

your name_____

The last four digits of your student ID_____

1. Find the area between the graphs of the two functions $f(x) = x^3 - 2x$ and $g(x) = x^2$.

2. a) Find the volume of the solid obtained by rotating around the y-axis the region bounded by the lines $y = 0$, $x = 1$, and $x = 4$ and by the graph of the function $y = \sqrt{x}$.
b) Find the volume of the solid obtained by rotating the same region around the line $y = -1$.

3. a) Recall that $\sinh x = \dfrac{e^x - e^{-x}}{2}$ and $\cosh x = \dfrac{e^x + e^{-x}}{2}$. Show that $\cosh^2 x - \sinh^2 x = 1$.
b) Show that the derivative of $\cosh x$ is $\sinh x$.

4. Find the sum of the geometric series $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

5. a) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \dfrac{(-1)^n (n+1) x^n}{2^n}$.

b) Test the convergence at the end points of the interval of convergence.

6. The differential equation

$$P'(t) = aP(t)\left(1 - \frac{P(t)}{k}\right)$$

is called "logistic" and it is frequently used to model population growth. The parameters a and k are positive. Explain why the maximum value of $P'(t)$, namely the maximum growth rate of the population, occurs when $P = \dfrac{k}{2}$.

7. Recall that the relation between the rectangular (or Cartesian) coordinates and the polar coordinates is

$$x = r\cos t, \quad y = r\sin t.$$

a) Find the Cartesian equation of the curve having polar equation $r = 2\cos t$.
b) Find the polar equation of the circle $(x-1)^2 + (y-2)^2 = 5$.

8. An investment manager determines that the growth of an investment $V(t)$ is proportional to $V^2(t)$. The initial value of the investment was \$ 10,000 and it took 10 years to reach \$ 20,000. How long will it take before the investment is worth \$ 50,000? Set up and solve a differential equation to answer this question.

Solution

1. Setting $f(x) = g(x)$ we obtain x=0, x=–1, and x=2. In [–1,0] f is larger than g and in [0,2] g is larger than f. Taking this into account we find that the area is 37/12.

2. a) Using rectangles parallel to the axis of rotation we find that the volume is given by $2\pi\int_1^4 x\sqrt{x}dx = 255\pi/4$.

b) Using rectangles perpendicular to the axix of rotation we find that the volume is given by $\pi\int_1^4 (x-1)dx = \frac{9\pi}{2}$.

3. a) We have

$$\left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2 = \left(\frac{e^x+e^{-x}}{2} + \frac{e^x-e^{-x}}{2}\right)\left(\frac{e^x+e^{-x}}{2} - \frac{e^x-e^{-x}}{2}\right) = e^x e^{-x} = e^0 = 1$$

b) Evident

4. Since the series starts at 1 and the ration is 2/3 we have that the sum is

$$\frac{1}{1-2/3} - 1 = 3 - 1 = 2$$

5. a) Using the ratio test for the absolute value we find r=2.

b) For x=2or x=–2 the general term does not go to 0. Hence the series does not converge.

6. The maximum value for the growth rate is taking place when the function $ax(1-\frac{x}{k})$ is largest. Using the derivative we find that this is taking place when $1-2x/k = 0$, i.e. x=k/2.

7. a) We have $\sqrt{x^2+y^2} = \frac{2x}{\sqrt{x^2+y^2}}$. Hence $x^2+y^2-2x = 0$.

b) We have $(r\cos t - 1)^2 + (r\sin t - 2)^2 = 4$. Expand and simplify to get $r^2 - 2r\cos t - 4r\sin t + 1 = 0$.