MATH 30, PRACTICE TEST I

Disclaimer: This practice test DOES NOT serve as an indication of the contents of the actual test. It only suggests a possible format.

Please print your name clearly!

Name:_____

Please show all your work, that is explain every step of your solution - it is your work, not the answer, that is being evaluated. When asked to prove a statement, make sure to provide reasoning behind each claim you are making in the process of the proof. The use of calculators or any other electronic devices is prohibited during the test. You are also not allowed to use any study materials except for those provided to you during the test. Cheating is strictly prohibited and will be prosecuted. Good luck!

Problem 1 (20 points). Find inverses of the following functions, if they exist. For each inverse, specify the domain and range.

a) - (10 points) $f(x) = \sqrt{x} - 7$

b) - (10 points) $g(x) = x^4 - 5$, $x \leq 0$

Problem 2 (20 points). Find the specified values of inverse trigonometric functions.

a) - (10 points) $\tan^{-1}\left(\tan\left(\frac{27\pi}{3}\right)\right)$

b) - (10 points) $\sin^{-1}\left(\sin(\pi)+\cos\left(\frac{\pi}{3}\right)\right)$

Problem 3 (20 points). Find the domains of the following functions.

a) - (10 points) $f(x) = e^{-\sec x}$

b) - (10 points) $g(x) = \ln(\sin x)$

Problem 4 (30 points). Compute the following limits or show that they do not exist.

a) - (10 points) $\lim_{x\to 4} \frac{x^2-2x-8}{x^2-x-12}$

b) - (10 points) $\lim_{x\to 0^+} \tan^{-1}(\ln x)$

c) - (10 points) $\lim_{x\to\infty}(e^{-x}+\sin x)$

Problem 5 (30 points). Determine for which values of x the following functions are continuous.

a) - (10 points) $f(x) = \frac{x-2}{x^2+2}$.

b) - (10 points) $g(x) = \frac{\sqrt{x}}{\sin(x)}$.

c) - (10 points) $h(x) = \begin{cases} 3x+2 & \text{if } x < 2 \\ x^3 & \text{if } 2 \leq x \leq 3 \\ 3x^2+x-1 & \text{if } x > 3 \end{cases}$

Problem 6 (20 points). Use the formal ε / δ definition of the limit to prove that

$$\lim_{x \to -\frac{1}{3}} \left(\frac{12x^2 - 5x - 3}{3x + 1} \right) = -\frac{13}{3}.$$

Problem 7 (20 points). Let

$$f(x) = \sqrt{x}.$$

Use the limit definition of the derivative to find the function $f'(x)$ and specify its domain.