MATH 31-1, SPRING 2021, PRACTICE TEST I

Disclaimer: This practice test DOES NOT serve as an indication of the contents of the actual test. It only suggests a possible format.

Please print your name clearly!

Name:_____

Please show all your work, that is explain every step of your solution - it is your work, not the answer, that is being evaluated. When asked to prove a statement, make sure to provide reasoning behind each claim you are making in the process of the proof. The use of calculators or any other electronic devices is prohibited during the test. You are also not allowed to use any study materials except for those provided to you during the test. Cheating is strictly prohibited, and will be prosecuted. Good luck!

Problem 1. Let $f(x) = \sin x$ and $g(x) = -\cos x$.

a) Find the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$ on the interval from $\frac{\pi}{4}$ to $\frac{\pi}{3}$.

b) Find the volume of the solid obtained by rotating the region bounded by $h(x) = \frac{f(x)}{g(x)}$, x-axis, and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ about the x-axis.

Problem 2. Evaluate the following integrals:

a)

$$\int \sin^3 x \cos^2 x \; dx$$

b)

$$\int \tan^4 x \sec^4 x \; dx$$

c)

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} \; dx$$

Problem 3. Evaluate the following definite integrals:

a)
$$\int_0^1 e^x \cos x \; dx$$

b)
$$\int_0^1 \sqrt{4 - x^2} \; dx$$

c)
$$\int_1^2 x \ln x \; dx$$

d)
$$\int_0^1 \sin(\pi\sqrt{x}) \; dx$$

Problem 4. The natural length of a spring is 10 (do not worry about units in this problem). A force of 40 is applied to stretch the spring to the length of 20. Suppose that a force is continuously applied to stretch this spring from its natural length to the length of 30.

a) What is the average value of this force on the given interval?

b) Is this average value of the force assumed at some point on the interval? Explain why or why not. If it is assumed somewhere, find the exact length of the spring at which the force is equal to this average value.

Problem 5. Consider the region R bounded by the curves $y = \sqrt{x}$ and $y = x^2$. Let S be the solid object obtained by rotating this region about the y-axis.

a) Find the volume of S using the washer method.
b) Find the volume of S using the method of cylindrical shells.