



Claremont Graduate University, Fall 2022 MATH 389E: Topics in Discrete Optimization

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Time: Thursdays, 1:00 – 3:50 pm

Prerequisites: In-depth knowledge of linear algebra, as well as general familiarity with real analysis and abstract algebra. I am happy to talk to anybody interested in this course, and in particular to discuss if their background is sufficient.

Course description: This course will focus on several classical optimization problems and spend a part of the semester on each of them. The specific problems considered may include:

1. ***The Integer Knapsack Problem:*** given a collection of objects with assigned weight and cost, maximize the cost function while keeping the weight under the specified threshold. In addition to its intrinsic mathematical significance, this problem often comes up in resource allocation.
2. ***The Frobenius Problem:*** given a collection of relatively prime positive integers, find the largest positive integer that cannot be represented as their nonnegative integer linear combination. This problem is closely related to the integer knapsack, and (same as integer knapsack) is known to be NP-hard in general.
3. ***The Main Problem of Coding Theory:*** maximize the error-correcting capability of a linear code while keeping its codeword length bounded. This problem is central in the study of accurate data transmission over potentially noisy channels.
4. ***Optimization Problems on Lattices:*** optimize packing density, covering thickness and kissing number of a Euclidean lattice in n dimensions. This is the main problem of lattice theory, a branch of mathematics at the intersection of number theory and discrete geometry. In addition to its theoretical value, it has numerous applications, for instance in digital and wireless communications.
5. ***Coherence Minimization on Euclidean Frames:*** find frames (overdetermined spanning sets) in Euclidean vector spaces of large cardinality and small coherence. Such frames allow for sufficiently fast data transmission with efficient erasure-recovery capabilities.

These problems have many features in common, in particular all of them have geometric interpretation as well as applications in digital communications. The goal of this course would be to give an introduction to these deep problems, to discuss their unified geometric framework, and to indicate some applications.

Registration is open to students from all of the Claremont Colleges, and I am happy to talk to anyone interested in this course!