Name................
Math Analysis I
HW 2
Due 02/07/2013

a) Let $f : [\frac{\pi}{2}, \frac{3\pi}{2}] \to [-1, 1]$ be given by $f(x) = \text{sinx}$. True or False : f is a bijection, and its inverse function is arcsinx.

b) Find $f(E)$ and $f^{-1}(E)$ for $f(x) = 2 - 3x$ and $E = (-1, 2)$

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2. Suppose $f : A \to B$ and $g : B \to C$ are functions show that

a) If both f and g are one-to-one, then $g \circ f$ is one-to-one.

b) If both f and g are onto, then $g \circ f$ is onto.

c) If both f and g are bijection, then $g \circ f$ is bijection.

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3. For a function $f : X \longrightarrow Y$, show that the following statements are equivalent.

a) f is one-to-one

b) $f(A \cap B) = f(A) \cap f(B)$ holds for all $A, B \in \mathcal{P}(X)$

Hint: For $a) \Rightarrow b)$ you can assume $f(A \cap B) \subseteq f(A) \cap f(B)$.
For $b) \Rightarrow a)$ consider $A = \{a\}$ and $B = \{b\}$.

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4. For an arbitrary function $f : X \longrightarrow Y$, prove the following identities:

a) $f^{-1}\left(\bigcup_{i\in I} B_i\right) = \bigcup_{i\in I} f^{-1}(B_i)$

b) $f^{-1}\left(\bigcap_{i\in I} B_i\right) = \bigcap_{i\in I} f^{-1}(B_i)$

c) $f^{-1}(B^c) = [f^{-1}(B)]^c$

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5.

a) Show that the set of irrational numbers in $(0, 1)$ is not countable.

b) Show that any nonempty subset of a countable set is finite or countable.

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6. An algebraic number is a root of a polynomial, whose coefficients are rational. Show that the set of all algebraic numbers is countable.

Hint: Use the Fundamental Theorem of Algebra: A polynomial of degree n can have at most n roots. You may also need the fact that countable union of countable sets is countable.

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7. Given any set A show that there does **not** exist a function $f : A \to \mathcal{P}(A)$ that is onto.

Hint: Prove by contradiction. Assume $f : A \to \mathcal{P}(A)$ is onto. Notice that f is a correspondence between a set and its power set. Therefore the assumption that f is onto means that every subset of A appears as $f(a)$ for some $a \in A$. To arrive at a contradiction, produce a subset $B \subseteq A$ that is not equal to $f(a)$ for any $a \in A$.