Math Analysis I
HW 1
Due 01/31/2013

1. For two sets A and B show that the following statements are equivalent.

a) $A \subseteq B$

b) $A \cup B = B$

c) $A \cap B = A$

Hint: Show that *a)* $\Rightarrow$ *b)*, *b)* $\Rightarrow$ *c)* and *c)* $\Rightarrow$ *a)*

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2. Establish the following set theoretic relations:

a) $A \cup B = B \cup A, \quad A \cap B = B \cap A$ (Commutativity)

b) $A \cup (B \cup C) = (A \cup B) \cup C, \quad A \cap (B \cap C) = (A \cap B) \cap C$ (Associativity)

c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributivity)

d) $A \subseteq B \iff B^c \subseteq A^c$

e) $A \setminus B = A \cap B^c$

f) $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ (De Morgan's laws)

Note that for $A \subseteq \mathbb{R}$,the complement of A, written A^c, refers to the set of all elements of $\mathbb{R}$ not in A. Thus,

$$A^c = \{x \epsilon \mathbb{R} : \ x \notin A\}$$

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3. Use the induction argument to prove that

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

for all natural numbers $n \geq 1$.

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4. Use the induction argument to prove that $n^3 + 5n$ is divisible by 6 for all natural numbers $n \geq 1$.

5. Let f, g be two functions defined from $\mathbb{R}$ into $\mathbb{R}$. Translate using quantifiers the following statements:

1. f is bounded above;
2. f is bounded;
3. f is even;
4. f is odd;
5. f is never equal to 0;
6. f is periodic;
7. f is increasing;
8. f is strictly increasing;
9. f is not the 0 function;
10. f does not have the same value at two different points;
11. f is less than g;
12. f is not less than g.

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6. Consider the four statements

$$\begin{array}{ll} (a) & \exists x \in \mathbb{R}\ \forall y \in \mathbb{R} \quad x + y > 0; \\ (b) & \forall x \in \mathbb{R}\ \exists y \in \mathbb{R} \quad x + y > 0; \\ (c) & \forall x \in \mathbb{R}\ \forall y \in \mathbb{R} \quad x + y > 0; \\ (d) & \exists x \in \mathbb{R}\ \forall y \in \mathbb{R} \quad y^2 > x. \end{array}$$

1. Are the statements a, b, c, d true or false ?
2. Find their negations.

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7. Show by induction that if X is a finite set with n elements, then $\mathcal{P}(X)$, the power set of X (i.e. the set of subsets of X), has 2^n elements.