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Math Analysis I
HW 1
Due 01/26/2012

1) For two sets A and B show that the following statements are equivalent.

a) $A \subseteq B$

b) $A \cup B = B$

c) $A \cap B = A$

Hint: Show that $a) \Rightarrow b)$, $b) \Rightarrow c)$ and $c) \Rightarrow a)$

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2) Give a simple description of each of the following sets

a) $\bigcap_{k \in \mathbb{N}} [-\frac{1}{k}, \frac{1}{k}]$

b) $\bigcup_{k \in \mathbb{N}} [-\frac{1}{k}, 0]$

c) $\bigcup_{k \in \mathbb{N}} [-k, \frac{1}{k})$

d) $\bigcap_{k \in \mathbb{N}} [-\frac{k-1}{k}, \frac{k+1}{k}]$

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3)
Establish the following set theoretic relations:

a) $A \cup B = B \cup A$, $A \cap B = B \cap A$ (Commutativity)

b) $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$ (Associativity)

c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributivity)

d) $A \subseteq B \iff B^c \subseteq A^c$

e) $A \setminus B = A \cap B^c$

f) $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ (De Morgan's laws)

Note that for $A \subseteq \mathbb{R}$,the complement of A, written A^c, refers to the set of all elements of $\mathbb{R}$ not in A. Thus,

$$A^c = \{x \epsilon \mathbb{R} : \ x \notin A\}$$

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4) Use the induction argument to prove that

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

for all natural numbers $n \geq 1$.

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5) Use the induction argument to prove that $n^3 + 5n$ is divisible by 6 for all natural numbers $n \geq 1$.

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6) Let f, g be two functions defined from $\mathbb{R}$ into $\mathbb{R}$. Translate using quantifiers the following statements:

a) f is bounded above;

b) f is bounded;

c) f is even;

d) f is odd;

e) f is never equal to 0;

f) f is periodic;

g) f is increasing;

h) f is strictly increasing;

i) f is not the 0 function;

j) f does not have the same value at two different points;

k) f is less than g;

l) f is not less than g.

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7) Consider the four statements

$$\begin{array}{ll} (a) & \exists x \in \mathbb{R}\ \forall y \in \mathbb{R} \quad x+y>0; \\ (b) & \forall x \in \mathbb{R}\ \exists y \in \mathbb{R} \quad x+y>0; \\ (c) & \forall x \in \mathbb{R}\ \forall y \in \mathbb{R} \quad x+y>0; \\ (d) & \exists x \in \mathbb{R}\ \forall y \in \mathbb{R} \quad y^2>x. \end{array}$$

a) Are the statements a, b, c, d true or false ?

b) Find their negations.

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8) Show by induction that if X is a finite set with n elements, then $\mathcal{P}(X)$, the power set of X (i.e. the set of subsets of X), has 2^n elements.

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