Your Name
Math Analysis II
HW 8
04/16/13

1. Investigate whether the system:

$$u(x, y, z) = x + xyz$$
$$v(x, y, z) = y + xy$$
$$w(x, y, z) = z + 2x + 3z^2$$

can be solved for x, y, z in terms of u, v, w near $(0, 0, 0)$.

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2.Discuss the solvability of

$$y + x + uv = 0$$
$$uxy + v = 0$$

for u, v in terms of x, y near $x = y = u = v = 0$ and check directly.

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3. Let

$$f(x) = x + 2x^2 sin(\frac{1}{x})$$

for $x \neq 0$ and $f(0) = 0$. Show that $f'(0) \neq 0$ but that f is not locally invertible near 0. Why does this not contradict the inverse function theorem?

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4. Show that if f is continuous, 1-1 mapping of a **compact** metric space X onto a metric space Y, then the inverse mapping f^{-1} defined on Y by

$$f^{-1}(f(x)) = x$$

for $x \in X$ is a continuous mapping of Y onto X.

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5. Define: $x : R^2 \to R$ by $x(r, \theta) = rcos\theta$ and define $y : R^2 \to R$ by $y(r, \theta) = rsin\theta$

1. Show that $\frac{\partial(x, y)}{\partial(r, \theta)}|_{(r_0.\theta_0)} = r_0$
2. When can we form a smooth inverse function $(r(x, y), \theta(x, y))$? Check directly and check by using the inverse function theorem.

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6. Let

$$F(x, y) = xy^2 - 2y + x^2 + 2$$

1. Check directly (without using implicit function theorem) to see where you can solve this equation for y in terms of x.
2. Check your answer in part a) if it agrees with the answer you expect from the implicit function theorem. Compute $\frac{dy}{dx}$.

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7. Discuss the solvability of the system

$$3x + 2y + z^2 + u + v^2 = 0$$

$$4x + 3y + u^2 + v + w + 2 = 0$$

$$x + z + w + u^2 + 2 = 0$$

for u, v, w in terms of x, y, z near $x = y = z = 0$, $u = v = 0$ and $w = -2$.

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8. Consider the equations

$$u(x, y) = \frac{x^4 + y^4}{x} \quad \text{and} \quad v(x, y) = sinx + cosy$$

a) Show that we can solve this system near $(\frac{\pi}{2}, \frac{\pi}{2})$ in terms of u and v.

b) Find $\frac{\partial x}{\partial v}$ and $\frac{\partial y}{\partial u}$.

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9. Take $n = m = 1$ in the implicit function theorem, and interpret the theorem graphically.

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10. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a map given by $f(x, y, z) = (x, y^3, z^5)$. Show that f has a global inverse g, despite the fact that $Df(0)$ is singular. What does this imply the differentiability of g at 0?

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