Prof. Asuman Aksoy
Math Analysis I
HW 4
Due 02/16/2012

1) For each of the following sets S, find $\sup(S)$, $\inf(S)$ if they exist:

a) $\{.3, .33, .333, \cdots\}$

b) $\{\frac{1}{n} : \ n, \text{ an integer}, n > 0\}$

c) $\{\frac{-1}{n} : \ n, \text{ an integer}, n > 0\}$

d) $\{x \in \mathbb{R} : \ x^2 < 5\}$

e) $\{x \in \mathbb{R} : \ x^2 > 5\}$

■

2) Let S and T are nonempty bounded subsets of $\mathbb{R}$with If $S \subset T$. Prove that:

$$\inf T \leq \inf S \leq \sup S \leq \sup T$$

■

3) Let$\{I_n\}$ be a decreasing sequence of nonempty closed intervals in $\mathbb{R}$, i.e. $I_{n+1} \subset I_n$ for all $n \geq 1$. Show that $\bigcap_{n \geq 1} I_n$ is a nonempty closed interval. When is this intersection is a single point?

■

4) Suppose (x_n) and (y_n) are Cauchy sequences, then show that

a) $(x_n + y_n)$ is a Cauchy sequence.

b) $(x_n y_n)$ is a Cauchy sequence.

■

5) Show that if a subsequence of a Cauchy sequence converges to x, then the sequence itself converges to x.

■

6) Let x and y be two different real numbers. Show that there exist a neighborhood X of x and a neighborhood Y of y such that $X \cap Y = \emptyset$.

Hint: You must choose your $\varepsilon > 0$ so that the intersection of $X = (x - \varepsilon, x + \varepsilon)$ and $Y = (y - \varepsilon, y + \varepsilon)$ is empty.

■

7) If α and β are in $\mathbb{R}$ and $\alpha < \beta$, then every sequence of points in the interval

$$[\alpha, \beta] = \{x : \ \alpha \leq x \leq \beta\}$$

has a subsequence that converges to some point in $[\alpha, \beta]$

■

8) (**Cesaro Average**) Let $\{x_n\}$ be a real sequence which converges to l. Show that the sequence

$$y_n = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

also converges to l. What about the converse?

Hint: Notice that

$$y_n - l = \frac{x_1 + x_2 + \cdots + x_n}{n} - l = \frac{(x_1 - l) + (x_2 - l) + \cdots + (x_n - l)}{n}.$$

For the converse take $x_n = (-1)^n$.