ANTC Seminar 29 March, 2011

Partially Recursive Acceptance Rejection

How do I roll a 7-sided die?

Repeat
  Let X be roll of 10-sided die
Until $X \in \{1, \ldots, 7\}$.

$\mathbb{P}(X = 4) = 1/7$

$$= \frac{1}{10} + \frac{3}{10} \cdot \frac{1}{10} + \left(\frac{3}{10}\right)^2 \left(\frac{1}{10}\right) + \ldots$$

7 1st roll; 8, 9, 10 1st roll; 7 second roll

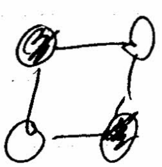
$$= \frac{1/10}{1 - 3/10} = \frac{1}{7}.$$

High dimensional Problem, doesn't work so well

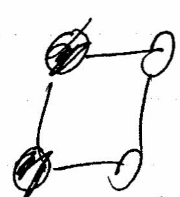
Put Up on Board

Independent Sets

Def: An *independent set* of a graph is a subset of nodes where no two nodes in subset are connected by an edge.



ind set



not an ind set.

Goal: • Generate samples from weighted ind sets

$$\pi_\lambda(A) = \frac{\lambda^{\#A}}{Z}.$$

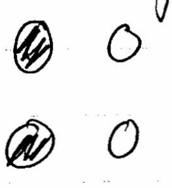
• Can be used to approximate Z

Applications: 1) Z is #P complete
2) Simple model of repulsive points (cities, trees).

Basic Acceptance rejection

1) Repeat
2) $A \leftarrow \emptyset$
3) For all $v \in V$
4) Add v to A w/ prob. $\lambda/1+\lambda$
5) Until A is an independent set

4), 5) Treat graph like it had no edges.



reject

$$\mathbb{P}(A) = \left(\frac{\lambda}{1+\lambda}\right)^{\#A}\left(\frac{1}{1+\lambda}\right)^{\#V-\#A}$$

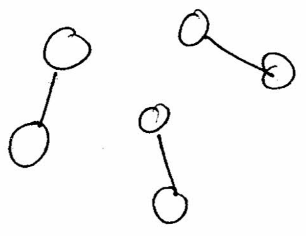
$$= \underbrace{\left(\frac{1}{1+\lambda}\right)^{\#V}}_{\text{Constant}} \cdot \underbrace{\lambda^{\#(A)}}_{\propto \pi_\lambda}$$

Good enough for acceptance rejection.

Problem: Chance of acceptance very low

every edge, chance ~~accepting~~ failure $\frac{\lambda^2}{(1+\lambda)^2}$

So for multiple edges (say α ind. edges)



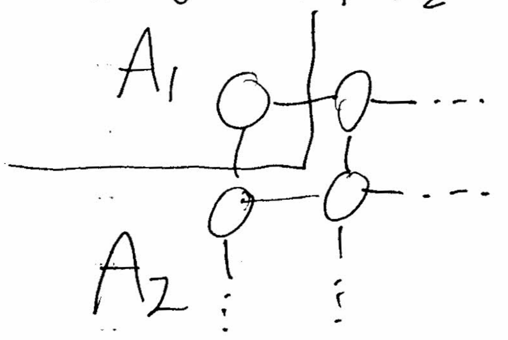
$\left(1-\frac{\lambda^2}{(1+\lambda)^2}\right)^{\alpha}$ is upper bound on Change of success.

Solution PRAR

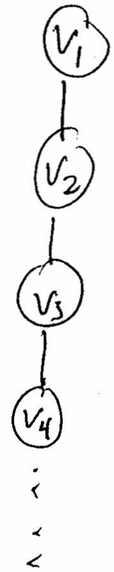
Recursive Acceptance Rejection

Repeat
- Pick $v \in V$
- Let $A_1 \leftarrow \{v\}$ w/ prob. $\lambda/1+\lambda$, otherwise $A_1 \leftarrow \{\}$
- Draw A_2 from ~~graph~~ π_λ on graph induced by $V \setminus \{v\}$.

Until $A_1 \cup A_2$ is an independent set

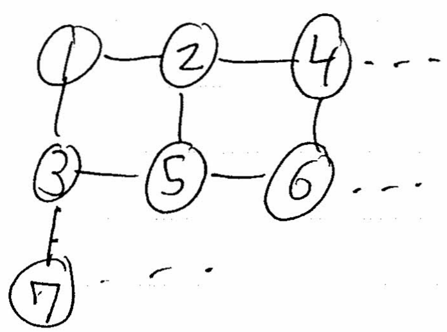


Recursion Tree



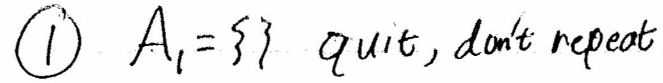
Still Exponential

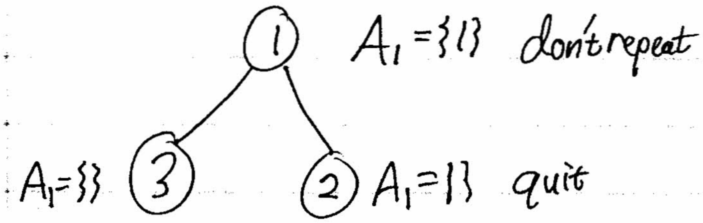
PRAR comes from simple idea that you don't need all recursion

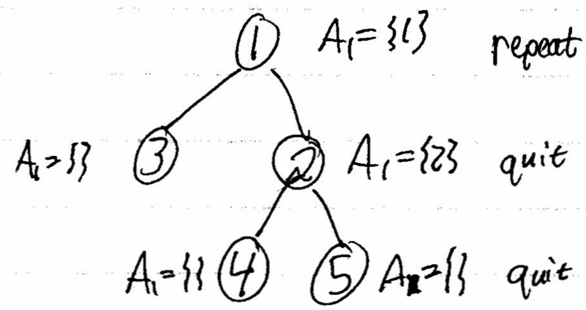


Repeat
  Pick any $v \in V$
  ~~Make~~ Set $A_1 \leftarrow \{v\}$ w/ prob. $\frac{\lambda}{1+\lambda}$, else set $A_1 \leftarrow \{\}$.
  If $A_1 = \{v\}$
    Let ~~me~~ $N \leftarrow$ neighbors of v
Recursion $\longrightarrow$ Find out whether nodes in N are part of ind. set.
Until $A_1 = \{\}$ or no nodes in N part of ind. set.

Typical Recursion Tree







Running time: Does tree grow or shrink on average?

T = mean time to resolve tree w/ one node

$$T \le \underbrace{\frac{1}{1+\lambda}}_{\text{chance of } \{\}} \cdot (1) + \frac{\lambda}{1+\lambda}\left[T(\underbrace{\Delta}_{\text{max degree of graph}}-1) + T \right]$$

might reject and do original node over

$$(1+\lambda)\,T \le 1 + \lambda(\Delta)\,T$$

$$\left[1+\lambda-\lambda(\Delta)\right]T = 1$$

$$T \le \frac{1}{1-\lambda(\Delta-1)} \qquad \text{for } 1-\lambda(\Delta-1) \ge 0$$

$$\lambda \le \frac{1}{\Delta-1}.$$

PRAR not only method for generating independent sets

- Bounding chains [H '99]
- Randomness Recycler [Fill, H. 2001]

Which is better?

Bounding chains $\frac{2}{\Delta - 2}$

RR $\frac{3/2}{\Delta - 1}$

PRAR $\frac{1}{\Delta - 1}$

Pessimist: Don't waste your time

Optimist: Opportunity for improvement

Conj.: Provably poly time for $\lambda < \frac{e}{\Delta - 1}$