Public good: optimal provision & Samuelson equation

Definition – the provision of a public good is socially optimal where the sum of individual marginal rates of substitution is equal to the social marginal cost

Source: Gruber J. (2016) *Public Finance & Public Policy*

Intuition – Everyone in society can freely use a public good, so all must share the same quantity. We find the amount by looking at how much each person in society is willing to give up buying other things in favor of having the public good. The total of these tradeoffs is our collective preference. We can also figure out how much each person will pay for the good at various quantities. In this case, the shared willingness to pay is the sum of the values at each quantity. We must compare that to the costs to supply one more unit of the public good. The level we share must equal the the cost of supplying one more unit of the good.

Mathematical / Technical –

- The marginal rate of substitution, $MRS_{S^*,c}$, is the willingness to trade off private consumption to gain some public good.
- The sum of these tradeoffs is represented by the Samuelson equation, with i individuals:

$$\sum_i MRS^i_{S^*,c}(Q^*) = MC(Q^*)$$

$$MRS^1_{S^*,C} + MRS^2_{S^*,C} + MRS^3_{S^*,C} = MC$$

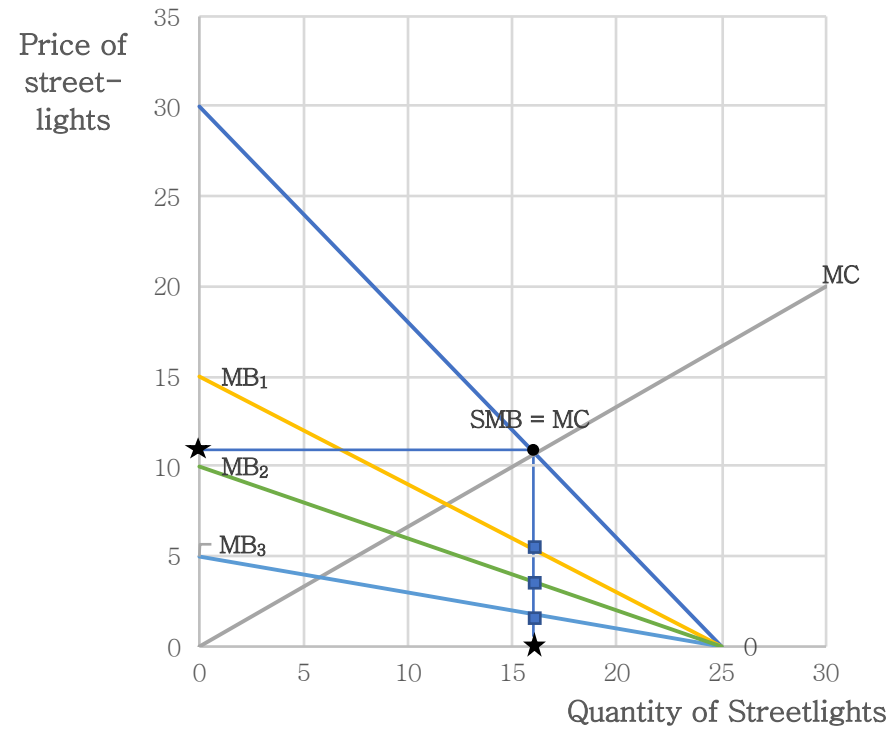
- The ***sum*** of each person's personal tradeoffs between private consumption and the public good must equal the MC.
- The MRS is given by the slope of the indifference curve for private versus public consumption for each utility maximizing individual. As the prices and quantities change, the slope changes, yielding a demand curve.
- This demand curve can be represented as in inverse, with price as a function of quantity. This represents how much an individual is willing to pay for various levels of the public good.
- Therefore, the Samuelson can also be written in terms of marginal benefits, with i individuals:

$$SMB(Q^*) = \sum_i MB_i(Q^*) = MC\,(Q^*)$$

$$MB_1 + MB_2 + MB_3 = MC$$

- Thus, the social marginal benefit (SMB) can be measured by the ***sum*** of everyone's individual willingness to pay for various quantities of the public good.
- In both equations, Q^* is the optimal provision that satisfies the conditions.
- The optimal provision of a public good is where the SMB is equal to the marginal cost (MC). The MC is the cost associated with the production of an additional unit of a public good.
- Each person must for pay their share of the public good. Their shares are proportional to their contribution to the SMB through their MB or MRS.

Graphical – optimal shares, public good



The individual willingness to pay for streetlights are MB_1, MB_2, MB_3 and the sum of these is equal to the SMB. A dot denotes where the SMB is equal to MC. The stars denote the socially optimal price and quantities of streetlights and squares denote individual shares.

Real-world aspects – A large issue with optimal provision is that it is almost impossible to make people pay their shares. It is difficult to collect different amounts from everyone because it can seem unfair that some people pay more for the same amount go public good. Another issue involves estimating individual demand for public goods. Think about this: how much money would you give to have clean drinking water? People may not know their own willingness to pay for a public good like clean air, clean drinking water, education, and national defense. One example is charitable giving. For charitable giving, individuals can pay their share by donating various amounts of money towards a public good they want more of.

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Practice questions

1. Suppose there are 3 people in a society that consume a public good. Their inverse demands are
$$MB_1 = 50 - G$$
$$MB_2 = 100 - G$$
$$MB_3 = 150 - G$$
where G is the units and p is price. The marginal cost for the good is \$12. Using the Samuelson equation, find the optimal provision of the public good. Explain why the Samuelson condition results in the optimal provision.

2. Think of public goods you encounter everyday. Why might the government have difficulty collecting private funds for these public goods? Can you think of ways to overcome this?

3. Using the graph above, find the optimal provision (price and quantity) of streetlights. How much does each person spend on streetlights?

4. Why can the sums of both the MRS and the MB of a public good for every person in a society be used to estimate the social marginal benefit?

Numerical solutions: **1.** 22 **3.** P^* = \$11, Q^* = 16, $Person_1$ = \$5, $Person_2$ = \$4, $Person_3$ = \$2.