Majority voting & median voter theorem

Definition –
Majority voting will yield the outcome preferred by the median voter if preferences are single peaked.

Source: Gruber, J. (2019) *Public Finance & Public Policy*

Intuition –
Say five people are making a group choice between two goods. If everyone votes, the outcome that has the most votes wins. Suppose that persons *A* and *B* really want good *X*, but persons *C, D* and *E* really want good *Y*, the group will choose *Y* because more people want it. Interestingly, because *C* is the "middle" or median voter, if you just ask Person *C* instead of conducting a majority vote, the outcome is the same.

Mathematical / Technical

- **Majority voting** is often the mechanism by which a group makes a choice: defined by the plurality, the largest coalition.
- For a single-dimension, bi-model choice (*yes/no*, candidate *A/B*), one side will get 50% or more of the vote. The range of the dimension a runs from a_{min} to a_{max}.
- Each person has a personal distribution of utility levels over the dimension, the distribution describes their preference.
- The peak is highest point, a person's most favorable choice. If a person a only one peak, their preferences are single-peaked. If a person has many peaks, like a mountain, their preferences are multi-peaked.
- In aggregate, the voters can be sorted by their peaks. Let $a_1 < \cdots < a_I$ be the peaks of individuals $1, \ldots, I$, sorted from a_{min} to a_{max}.
- The median voter is the person whose peak, a_{median}, is the is in middle of the sorted list.
- **Median voter theorem**: when all voters have single-peaked preference, then if only the median voter is asked their preferences, the policy outcome is the same as a popular election.

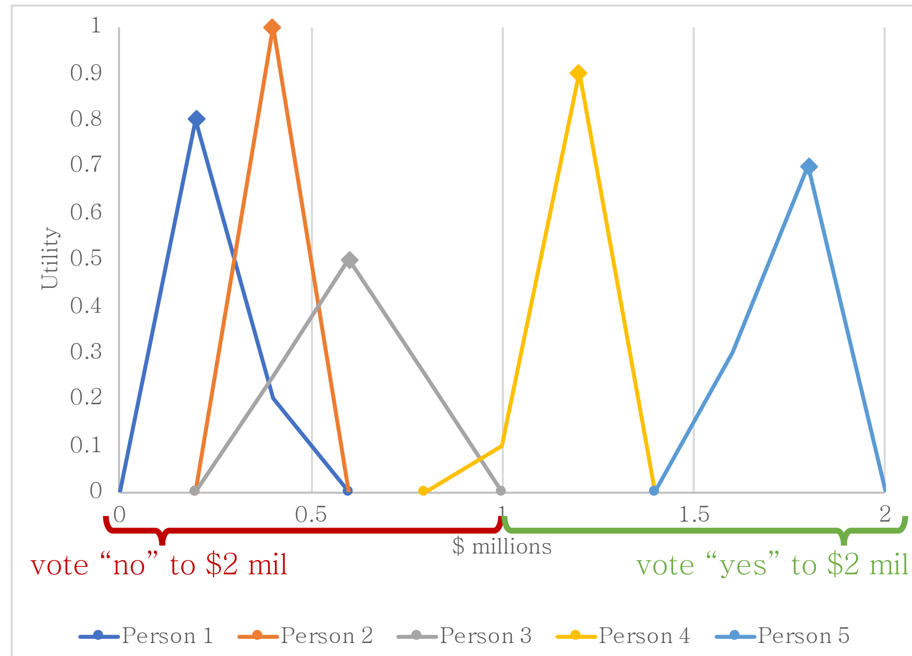
Proof

- Suppose there is a vote between two options: a_{median} and a^*, with $a_{median} < a^*$. a_{median} wins because $i = 1, \ldots, median$ all prefer a_{median} to a^* because they all have decreasing preferences for a beyond a_{median}.
- Symmetrically, a_{median} wins against a^* if $a^* < a_{median}$ because $i = median, \ldots, I$ prefer a_{median} to a^*. ■

Real-world aspects –
Median voter outcome from majority voting is useful and a hugely influential result in the political economy literature. Yet, this theory does not always hold, due to its unrealistic assumptions, such as politicians having no ideology, no lobbying, and having full information. A recent study determines how close the theorem is to reality: they calculate the degree of difference between the policy outcome and median voter choice. This divergence indicates the level of polarization and electoral uncertainty, among other things.

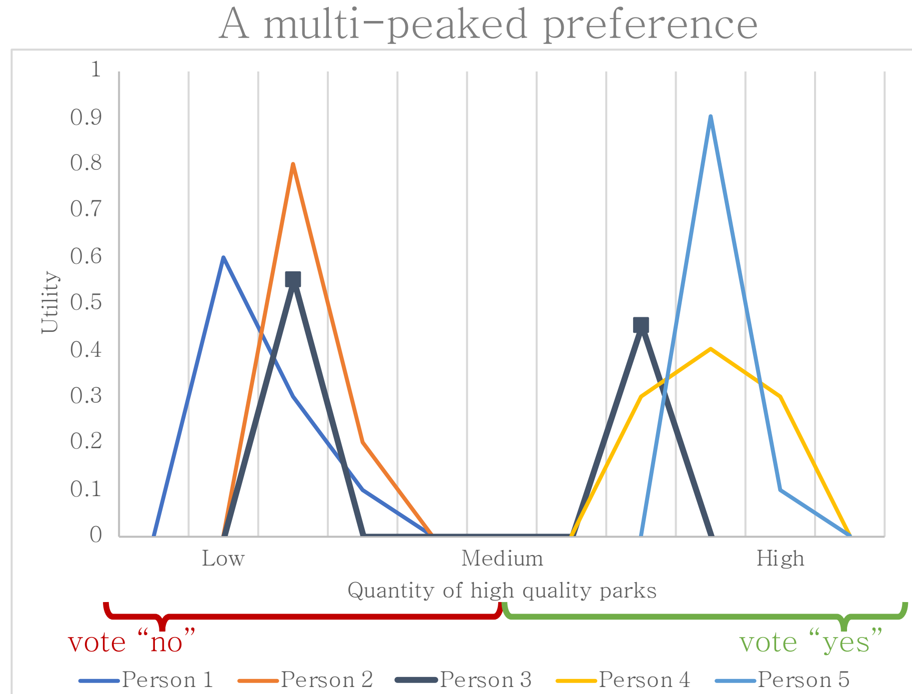
Source: Cukierman, A., Spiegel, Y. (2003) *Journal of Econ. & Politics*

Graphical – Single-peaked preferences



The Median Voter Theorem assumes that each person's preferences is each single peaked. Here is a vote for $2 million in spending. Person 3 is in the middle and tipped toward "no," so "no" will win. If person 3's peak were more than $1 million, the vote would be tipped to "yes."

A multi-peaked preference



A person's preferences are not always single peaked: A voter, like person 3, may have two very distinct options that they like. These situations undermine the model because person 3 flip-flops: no outcome wins.

Practice questions

1. An election coming up: on the ballot is an initiative to put $2 million towards cleaning up the local hiking trails. The community has 1,000 residents, and 300 of those residents do not want their taxpayer dollars going to renovating the trails. The other 400 people do want the renovations. The other 200 have little preference either way. Where does the median voter lie? Which group gets the policy they want?
2. The second graph depicts the preferences of residents in a different city, where there is an initiative to expand the number of high-quality parks. Imagine around 30% of voters have enough money to buy a membership for a country club. Why might they have a two peaked preference? How does this affect the median voter theorem?
3. A preschool class of five children is deciding how many class fish to get. The choices are 5 or 10 fish. The peaks of their preferences are as follows: $a_A \rightarrow 10$, $a_B \rightarrow 6$, $a_C \rightarrow 2$, $a_D \rightarrow 8$, $a_E \rightarrow 4$. Which student is the median voter? How many fish will the class get? How many fish would the median child need to want to flip the vote?

Numerical solutions: **1.** Renovators. **3.** Student B, 5 fish; ≥8 fish.