Gerrymandering in State Legislatures: Frictions from Axiomatic Bargaining

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Theories of partisan redistricting postulate unitary actors maximizing their party’s expected seat share. Yet the partition of a fixed supply of friendly voters necessarily implies a tragedy of the commons. We recast partisan redistricting as a bargaining game among the sitting representatives of the party controlling the map. The status quo is the threat point, explaining why changes are frequently minor. This bargaining framework implies that highly competitive districts will receive more help from redistricting if they are already represented by the party in charge. Employing a regression discontinuity design with precinct level data, we find support for this prediction.

I. Introduction

While the study of the boundaries of US Congressional districts has a long history, public interest has recently turned to US Statehouses. This is, in part, because a gridlocked US Congress has shifted legislative activity to the states and, in part, because it has been recognized that control of state legislatures enables a party to draw congressional district boundaries.¹ Recent attention to this longstanding truth seems to have been the result of the extreme success of a deliberate and well-funded Republican strategy to capture state legislatures in 2010 so as to control much of the subsequent decennial redistricting in 2011 (Daley 2016). Not only did Republican majorities in state legislatures draw the district lines for their Congressional delegations, they also drew district lines for their own state legislatures, potentially entrenching this source of political power, to the chagrin of Democrats who have mounted legal challenges.

While attention paid to state redistricting has waxed in recent years, it is not clear whether we have the proper positive model to explain the districts that emerge under the present system. Theories of partisan gerrymandering² based on explicit optimization typically assume a unitary actor free from the constraints

¹It also multiplies the number of maps under study from 1 to 99, which is a boon to statistical analysis.
²Partisan gerrymandering is the drawing of districts in such a way as to maximize the likelihood of or size of a majority for the party controlling the process. Bipartisan gerrymandering—when neither party has a monopoly on control—purportedly results in the drawing of districts in such a way as to protect incumbents of both parties.
of history. Some individual is presumed to draw the lines subject only to the
constraint of equal population and to a degree of uncertainty over the future
partisan preferences of voters.

Yet parties are not unitary. Members face a complex combination of indivi-
dual and collective goals. The partitioning of a fixed electorate with limited
coopartisans implies a dilemma for any individual member. Maximizing expected
party seat share entails transferring friendly voters from safe districts, where their
marginal effect on probability of victory is low, to competitive districts where
their marginal effect is high. Members in relatively safe districts are thus cross-
pressured. Improving the party’s expected seat share may require they reduce
their own probability of reelection. Even members who would receive an influx
of friendly voters under the optimal-seat-share scheme would likely prefer more
help than the optimal scheme would allocate them. It is a common pool problem
from which the map that maximizes expected seat share is unlikely to emerge.

We introduce a new model of gerrymandering that reflects this common pool
problem. We take as our starting point the model of Gul and Pesendorfer (2010)
which postulates that mappers observe vote choice from which they must infer
voters' true preferences. Mappers are then constrained to rearrange blocks of
voters whose average partisan composition is inferred from prior voting data.
This approach has the advantage of being both micro-founded and easily brought
to available precinct-level data. But instead of presuming that the map is designed
by a unitary actor to maximize expected seat share, we postulate that the map
is the product of a bargaining process among the sitting legislators.

While most states pass the new map through standard legislative procedure,
the majority party typically introduces a map that has already been agreed upon
amongst its members. Written accounts of the intra-party bargaining process
are unavailable, making it challenging to write a structural model of the relevant
bargaining game. Thus we choose an axiomatic approach to bargaining, the
Nash bargaining solution, in which all seated legislators of the party in power
are represented in the bargain. Legislators balance two considerations: they wish
to maximize their own chance of reelection and they wish to maximize the seat
share of their party. From a holistic perspective, districts that are already held
are represented in the bargain through both terms; they count toward party seat
share and one particular actor in the bargain—the current holder of the seat—
places extra weight on the party winning that particular seat.3 By contrast,
districts held by the other party are part of the expected seat share calculation
but do not have a specific advocate at the bargaining table. As a result, friendly
voters are, compared to the seat-share optimizing level, overly allocated to seats
already held, resulting in a departure from the optimal map.

Bargaining outcomes famously depend on the allocation that would prevail in
the event no agreement is struck. Cox and Katz (2002) convincingly argue the
importance of the reversionary outcome in the context of redistricting. Most state

3We have clearly abstracted from considerations of primary challenges.
constitutions set a deadline for drawing the lines. If the legislature cannot agree on a map before the deadline, the state might use either a backup commission or remand the issue to the state court. In the 2000 redistricting cycle, courts drew the lines in 11 of the 50 states (Levitt 2010, p28). In some cases, these backstops will share the partisan orientation of the legislature itself, but in most cases, the alternative is less partisan. As Levitt puts it, “Judges have little direct stake in the contours of particular legislative district lines, and may appoint individuals who similarly have little direct stake in the outcome of the redistricting process” (Levitt 2010, p28). To capture this lesser partisanship and the sense that caretaker commissions with little time and expertise are both less likely to radically redraw the map and likely to revise along lines orthogonal to partisan considerations, we presume that the disagreement point is the status quo.

The Nash bargaining solution maximizes the product of each player’s surplus beyond the disagreement point. This results in relatively few changes being made from the previous map as any change must deliver a Pareto improvement among the majority-party representatives. Thus while existing unitary-actor theories are entirely without inertia, results from our bargaining procedure are history-dependent, with the previous map acting as an important reference point. We believe this is important to explain what seems to be a high degree of inertia in the lines. One of the few recent descriptions of the process suggests that such universal involvement is near the truth. Discussing the 2010 redistricting process in Wisconsin, New York Times Magazine writer Emily Bazelon noted, “Nearly all of the 79 Republicans in the Wisconsin [state] Senate and Assembly made a similar trip to the map room…” to see the changes proposed for their district, strongly suggesting that all sitting majority party legislators participate in the process and make sure their voices are heard.4

One of the axioms of the Nash solution is symmetry, which could be questioned on the grounds that more senior members of the chamber might have better connections and more accumulated favors to call in and thus achieve a more favorable position in the bargain. In response, we note that more senior members are also more likely to be in relatively safe seats, from which losing friendly voters is less costly. As a result, if one adds seniority to the model, the predicted relationship between seniority and voter allocation is of indeterminate sign and the two effects cannot be distinguished empirically. Thus we keep to the simple version for clarity.

The Nash bargaining solution delivers results that are intuitive yet offer new insight. We reaffirm that the party in charge will collectively reassign friendly voters from safe districts to competitive districts. This acts to rebalance the electorate towards the optimal crack-and-pack solution. However, there are two important differences. First, this process is limited by the necessity of delivering any particular representative a positive net utility. Thus even the safest district currently

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held by the party will become only modestly more competitive before the holder of that district finds that donating additional voters overturns the internalized benefits of a larger majority. Second, the process privileges districts that are already held by the majority party and thus represented in the bargaining process. Suppose there exist in the chamber two seats whose electorates each appear to be a knife-edge 50-50 split. In the most recent election prior to redrawing the maps, one of them just tipped for the opposition while the other just tipped for the party in power. From the perspective of maximizing seat share, they should both receive equivalent assistance. But because the latter is represented in the bargain, that district’s changes will be more favorable. Our model thus delivers an important testable implication that enables us to evaluate the suitability of the unitary actor assumption.

Using precinct-level data, we calculate the change in normal democratic vote that results for each district from the 2010 redistricting wave. This requires GIS work with precinct boundaries and district boundaries to figure out which precincts are reallocated. The change in a district’s normal democratic vote represents the degree to which the majority party map-makers are either improving or reducing the chances of their candidate’s victory. Restricting to those chambers with clear partisan control of the redistricting process, and using previous vote margin as the running variable, we conduct a regression discontinuity analysis to measure the bias toward districts currently held by the party in power. The results support our theory in several ways. First, there is a statistically significant bias toward currently held districts of approximately 2 percentage points. Second, this bias is larger in chambers that are uncompetitive, where we would expect individual reelection considerations to trump the diminished value of expanding an already large seat-share majority. Third, friendly voters are transferred on net from safe seats to competitive seats.

In short, there is renewed interest in redistricting including the degree to which partisan gerrymandering inhibits fair representation and the conditions under which such behavior is constrained. To answer these questions requires an accurate model of the process. We believe that the unitary actor assumption upon which existing theories are based exaggerates the ability of the majority party to coordinate to solve the common pool problem. We build a model based on bargaining among existing members of the majority party caucus which delivers a testable hypothesis for which we find support in data from state legislatures.

The remainder of our paper consists of six sections. First, we present a pair of examples to give a clearer picture of the seemingly missed opportunities that we see as we scrutinize district maps and which motivated this theory. Second, we

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5 Incumbent advantages would break the symmetry. The majority party would enjoy a higher probability of victory in the seat currently held and a lower probability of victory in the seat held by the opposition. So long as both seats remain within the set to be helped by redistricting (and given they are the 50-50 seats, they will so remain), this means the seat held by the opposition would, in a unitary model, receive more assistance rather than less. Thus the results of our regression discontinuity cannot be explained by incumbent advantage.
briefly review the two most prominent prior theories of gerrymandering. Third, we develop the model of map-making as a bargain. Fourth, we present our data. Fifth, we conduct our empirical work. Finally, we conclude with discussion of the implications and ideas for further work.

II. Examples

We present two examples chosen from states with unified control of the process by a single party and without preclearance requirements.

A. Michigan State Senate Districts 9 and 10

In the redistricting wave following the 2010 census, Republicans enjoyed unified control over the redistricting process in Michigan. Each state senate district comprised roughly 260,000 people and the Republican party controlled 26 of the 38 seats following the 2010 election. District 9, centered on the northern Detroit suburbs of Warren and St. Claire Shores, had been lost to the Democrats by a mere 9,000 votes in a relatively close election. Its neighbor to the North, district 10, had been won by a similarly close margin of 7,000 votes. District 9’s neighbor to the east, district 13, had been won by a more comfortable margin of 20,000 votes. And in the further outlying districts 11 and 12, neighbors of district 10, the Republicans had enjoyed secure victories by 31,000 and 20,000 votes respectively. A feasible local goal for a unitary actor would be to flip district 9 and secure district 10 via exchanges with districts 2, 11, 12, and 13. The strategy to a mapper with our data on normal democratic vote share by precinct (see Map 1) is pretty clear. There is no way to flip district 2, so one ought to further pack it via exchange with district 9. Specifically, take the more conservative lakeside Grosse Point neighborhood from district 2 in exchange for some of the bluer inland bits of district 9. Secondly, one ought to secure district 10 by exchanging voters with 11 (and possibly 12 and 13). For instance, widen that liberal foot of district 11 to include more of district 10’s local blue enclave in exchange for inland voters from district 11. Neither geography nor the local availability of co-partisans should inhibit this seat gain.

However, in the event, the mappers did not pursue this course. District 11 did send a large square of conservative precincts to district 10 but did not take any liberal voters in return. Instead, district 11 received the redder lakeside neighborhoods of District 9 by way of compensation, thus not materially altering the partisan makeup of the district. Meanwhile, district 10 offloaded many of its bluest precincts to district 9. In short, district 9 was written off and deliberately packed to shore up district 10 and districts 11, 12, and 13 made no sacrifices despite having won by comfortable margins. In the first election under the new map, district 10 was retained by 21,000 votes while district 9 was lost by 25,000 votes. Districts 11, 12, and 13 were all comfortably retained. It would seem that districts 11, 12, and 13 were unwilling to shade their margins of victory in an
Figure 1. Michigan State Senate District Map for the 2000s. The smallest geographic units are voting precincts. The darker outlines are the state senate districts, indicated with numeric labels. The change in hue from red to blue indicates variation in the normal democratic vote from Republican to Democrat. The shading from light to dark indicates population. The yellow numerals indicate the electoral margins of the mapping party in the most recent election. The yellow arrows are suggestions developed in the text.

We attempt to flip district 9. We are not claiming to have proven this to be an error—it is possible that this was the proper choice to maximize expected seat share given expected vote share volatility over the next decade. Our argument rests on the statistical case to be made later. We simply offer this and the following as examples of what such missed opportunities look like.

B. Maryland State Senate Districts 23 and 30

At the same time in Maryland, Democrats controlled the redistricting process as they had in the previous wave. Following the 2010 election, Democrats held a commanding 35 of the 47 seats, each of which represented about 120,000 people. District 23, encompassing the western edge of the DC suburbs, was so blue that the Democratic nominee ran uncontested. On the other hand, District 30, on the
shores of Chesapeake Bay, had been won by a mere 1,000 votes. Due to somewhat slow local population growth, in the new map, District 30 was expanded inland until it touched district 23. This should have given excellent opportunity to swap voters with the overwhelmingly safe district 23 so as to bolster district 30. But the opportunity wasn’t taken. As a result, in the subsequent election of 2014, district 23 was again uncontested while district 30 was retained by a narrow margin of 1000 votes. Meanwhile, a similar coastal district to the South, district 29, also went without help after a narrow 1000-vote victory in 2010. District 29 was lost to the opposition by a mere 5,000 votes in 2014. It would seem that district 23 was unwilling to adjust to help district 30, despite this clearly raising the expected seat share of the majority party.

Once again, we do not offer this example as definitive proof. The curious unwillingness to break into the DC suburbs to cover the redder bay communities
could be the result of a desire to keep together like-minded communities so as to improve the quality of representation. But it is also consistent with the unwillingness of incumbent legislators to lose their preferred voters in order to expand the caucus. And as we will argue, the call to expand the caucus is likely to be especially weak in chambers like the Maryland senate where the party drawing the map already enjoys a robust super-majority. Our fundamental point is that the assumption of maximization of expected seat share which underlies the current models of gerrymandering does not seem to do well in describing the maps produced.
III. Prior Models

There are two prominent prior theories of partisan gerrymandering. The first is the well-known crack-and-pack theory (Owen and Grofman 1988; Gul and Pesendorfer 2010). This theory predicts that the party drawing the lines will pack as many opposition voters as possible into a few districts, enabling the mapmaker to create a large number of districts with modest yet secure majorities. From a wasted-votes perspective (McGhee 2014), the excessive majorities in opposition districts waste far more votes than the modest majorities in supportive districts, allowing the mapmaker’s party to achieve seat-share in excess of vote-share. Crack-and-pack predicts large majorities in opposition districts and smaller majorities in government districts. It also predicts that voters in opposition districts will be relatively similar while voters in government districts will be more widely dispersed along the political spectrum.
By contrast, Friedman and Holden (2008) suggest that optimizing mapmakers will slice the partisan distribution of voters and match from the outside in. That is, a slice of the most implacable foes will be matched with a modestly larger slice of the most stalwart supporters in a single district. The map-maker will then slice the remaining voters in the same way, covering the most ardent foes with a slightly larger set of the most ardent supporters. And so on until the final district is a single lump of voters from near the center of the distribution. In each case, the slice of supporters is somewhat larger than the slice of foes so as to ensure the median is a friendly voter. The size of this overbite depends on the degree of volatility in voter preferences. Friedman and Holden show that, in the face of uncertainty over voters’ true preferences, this “slice-and-cover” improves the likelihood that districts set up to favor the mapmaker’s party will actually be won by the mapmaker’s party after preference-uncertainty is resolved.

Both FH and GP offer compelling micro-foundations in the form of voters with latent preferences about which parties receive signals. We have chosen to base our theory on the latter partly because our analysis of maps and reading of commentary suggests that crack-and-pack behavior is more widespread than slice-and-cover but also because the structure is more easily matched to existing data.

IV. Theory

A. The Model

In this section, we first adopt and briefly describe the stochastic median voter model of Gul and Pesendorfer (2010). Parties are presumed to have fixed positions. Voters have symmetric, single-peaked preferences over a unidimensional policy space admitting an ideal point, $x$. While the ideal point is unknown to the mapper, the mapper receives signals on the partisan affiliation of a voter. Republicans have ideal points drawn from the cumulative distribution $I_R$ while Democrats have ideal points drawn from $I_D$. Republican policy is fixed at $+1$ while Democratic policy is fixed at $-1$. Thus a Republican voter is one whose ideal point is greater than zero while a Democratic voter has an ideal point less than zero: $I_R(0) = 0; I_D(0) = 1$. It is assumed that $I_R$ is strictly increasing and convex on $[0, 1]$, has a median in $[0, 1]$, and is continuous. Symmetric conditions hold for $I_D$.

We draw attention to three substantive assumptions inherent to this setup. First, the median Republican voter is more moderate than the Republican platform (and likewise for Democrats). Second, the convexity assumption—which is crucial for a later result—implies the density of Republican voters increases along the interval $[0, 1]$ which implies the distribution of voters within a party is unimodal but the distribution of all voters across both parties is bimodal. Third, voters are classified according to whichever platform is closest to their ideal point. As this is a stochastic voting model, that correctly tracks the party the voter will
more commonly vote for. This accords with our focus—which Gul and Pesendorfer seem to share—on the information derived from precinct-level voting records rather than measures of actual party membership.

Voters’ utility depends on the distance between their ideal point and the platform of the party whose candidate is elected plus a valance term, \( v \), drawn from cumulative distribution \( L(\cdot) \). Positive values of \( v \) are presumed to favor the democratic candidate. Thus a voter with ideal point \( x \) receives utility \( u_R(x, v) = -|1 - x| - v \) if the Republican candidate is elected and \( u_D(x, v) = -|1 - x| + v \) if the Democratic candidate is elected. The voter will prefer the Republican candidate if and only if \( v < x \) for \( x \in [-1, 1] \). We assume \( L \) is strictly concave, continuous, and symmetric around 0.

The chamber consists of \( N \) districts, currently split between the two parties, \( R \) and \( D \), controlling \( N_R \) and \( N_D \) districts respectively. Without loss of generality, assume \( N_R > N_D \) such that Republicans are the majority party in control of the redistricting process. By observing the precinct level vote shares of each party, the mapper can construct the fraction of voters in each district that are affiliated with each party.\(^6\) Prior to the current round of redistricting, each district \( i \) is endowed with a set of voters characterized by a fraction \( p_i \) affiliated with party \( R \) (thus \( 1 - p_i \) affiliated with party \( D \)). Redistricting then consists of choosing the changes in the partisan alignment of voters for each district, \( \Delta p_i \), under the constraint that voter’s partisan affiliations cannot be altered, thus \( \sum \Delta p_i = 0 \). It is worth emphasizing that our focus on the change, \( \Delta p_i \), is deliberate in recognition that our bargaining process produces path-dependence as each new map is not laid down on a tabula rasa, but made in reference to the previous lines.

If \( \theta_i = p_i + \Delta p_i \) is the proportion of Republicans in district \( i \) on voting day, then the median voter’s ideal point, \( x_i(\theta_i) \), is that which solves: \( \theta_i I_R(x_i) + (1 - \theta_i) I_D(x_i) = \frac{1}{2} \). For each \( \theta_i \), there is a unique median, the median is strictly increasing in \( \theta_i \) and \( x_i(\frac{1}{2}) = 0 \). Because the Republican party wins if \( d_i < x_i(\theta_i) \), the probability a Republican wins the district is thus: \( f(\theta_i) = L(x_i(\theta_i)) \). This is what Gul and Pesendorfer call the District Outcome Function. Their Lemma 1 establishes that as the leading party’s support increases, its probability of winning increases but at a decreasing rate: \( f(\frac{1}{2}) = 0 \) and \( f' > 0, f'' < 0 \) while \( f(\theta) > \frac{1}{2} \).

Mapmaking consists of partitioning the set of precincts into \( N \) districts\(^7\). We presume that each member of the majority party has both office-holding and policy motives\(^8\) such that the utility of a member of the majority party from

\[^6\]This can be considered a noisy estimate based on a single signal from the most recent election, or it could be constructed in the manner of normal Democratic vote, by using information from several recent elections.

\[^7\]The role of precinct as the fundamental building block with which map-makers work can be seen clearly in The League of Dangerous Mapmakers by Robert Draper, The Atlantic October 2012 in which a well-known map consultant, in his presentation to legislators and staffers, “...warns legislators to resist the urge to overindulge, to snatch up every desirable precinct within reach...” [emphasis added].

\[^8\]Fenno (1973) ascribes to legislators three motives: getting reelected, achieving influence within Congress, and making “good public policy." We include the first of these explicitly in our utility function and the last is a function of securing a working majority for one’s party and thus also included here. The
district $i$ is

$$U_i = f(p_i + \Delta p_i) + \gamma M \left( \frac{1}{N} \sum_{i \in N} f(p_i + \Delta p_i) \right)$$

The first term is the member’s own probability of victory given the allocation of voters. This represents the office-holding motive. The second term represents the policy motive and the parameter $\gamma$ is the relative weighting. The second term centers on a function $M$, the argument of which is the expected seat share of the party drawing the map. The function $M$ represents the ability of the party to translate larger majorities into preferred policies. We assume $M' > 0$ implying that larger seat share translates monotonically into more preferred policies, but that $M'' < 0$ implying that the marginal value of an extra seat is declining in the size of the majority.  

We consider the case of cooperative bargaining among the members of the majority party and presume the Nash bargaining solution in which case the map produced is that which satisfies

$$\max_{\{\Delta p_i\}_{i=1}^N} \Pi_{i \in N_R} (U_i - d_i)$$

$$= \max_{\{\Delta p_i\}_{i=1}^N} \Pi_{i \in N_R} \left( f(p_i + \Delta p_i) + \gamma M \left( \frac{1}{N} \sum_{i \in N} f(p_i + \Delta p_i) \right) \right) - f(p_i)$$

s.t. $\sum_i \Delta p_i = 0$

where $d_i$ is the disagreement point for member $i$. Among potential axiomatic bargaining solutions, we prefer the Nash approach because we favor scale invariance of utility thereby ruling out other popular choices such as egalitarian or utilitarian solutions.

We choose to define the threat point as the current distribution of voters. Different states have different backup plans should the parties in charge fail to produce a map in time (Cox and Katz 2002); some remand the issue to commissions, some to courts. Frequently the delays necessitate using the previous map for the first election past the due date, or remand to non-partisan actors with little time for making big changes and little interest in making changes with systematic partisan effects. Hence our selection of the previous map as the result of breakdown.

Second is beyond the scope of this model.

9Micro-founding these assumptions would necessitate adding within-party heterogeneity in policy preferences and an explicit policy process, which are beyond the scope of this paper.
The first order conditions imply the following:

\[ (3) \quad [\Psi \frac{\gamma}{N} M'(\frac{1}{N} \sum_{i \in N} f(p_i + \Delta p_i))] f'(p_j + \Delta p_j) = \lambda, \forall j \in N_D \]

\[ (4) \quad [\Psi \frac{\gamma}{N} M'(\frac{1}{N} \sum_{i \in N} f(p_i + \Delta p_i)) + 1] f'(p_i + \Delta p_i) = \lambda, \forall j \in N_R \]

\[ (5) \quad \Psi = \sum_{l \in N_R} \Pi_{k \in N_R, k \neq l} (U_k - d_k) \]

The first set of conditions, obtaining for each of the seats currently held by the minority party, states that the increase in the probability that the majority party flips the district, \( f' \), times the value of an extra seat in passing policy, \( M' \), must equal the shadow price. The second set of conditions, obtaining for each of the seats currently held by the majority party, is similar but contains an extra +1 term inside the brackets indicating that the seat is valued not only for its effect on the majority, but also directly by the member who would enjoy holding office.

From these we can derive the following propositions:

**Proposition 1:** If \( p_i = p_j, i \in N_R, j \in N_D \), then \( \Delta p_i > \Delta p_j \). This follows directly from the first order conditions and the assumption on \( f'' \). It implies that two districts, each won by a razor thin margin, one held by the majority party and one held by the opposition, will be treated differently, with more help sent to defending the marginal district already held than sent to flip the opposition district. Intuitively, this is because the representative of the majority party district is represented in the bargain while the potential challenger in the opposition district is not. Thus the office-holding utility of the potential challenger is ignored in the collective bargain.

**Proposition 2:** This resulting inequality in **Proposition 1** will be stronger the larger the majority. This follows from the assumption about \( M'' \). Essentially, there is a tension between the office-holding motives and the policy-holding motives. The latter induces efficient direction of majority-party-aligned voters to where they add the greatest number of expected seats while the former directs them to the districts currently held by the majority party. A larger majority reduces the marginal value of an additional seat thereby tipping the balance toward shoring up existing seats.
Proposition 3: On the contrary, the resulting inequality in Proposition 1 will be weaker in chambers and parties that place greater weight on policy motives relative to office-holding motives (larger \( \gamma \)). This result follows directly from the first order condition.

Proposition 4: \( p_i < p_j, i, j \in N_A \), then \( \Delta p_i > \Delta p_j \)

Proposition 5: \( p_i > p_j, i, j \in N_B \), then \( \Delta p_i > \Delta p_j \)

These last two results suggest that districts which are more competitive will receive more aid. This reproduces the intuition developed by Winburn (2007). Again, this follows from the assumption about \( f'' \).

B. Simulation

To visualize propositions 1-5, we have numerically simulated our Nash bargaining solution. By plotting \( \Delta p_i \) against \( p_i \) we are able to visually depict the discontinuity predicted by our model. Conveniently, the case of the unitary actor is nested in our model allowing us to illustrate cleanly that the discontinuity from Proposition 1, which forms the heart of our empirical work, derives directly from relaxing that assumption. As the relative weight on policy increases, members with significant majority seats are willing to give up some of their supporters to assist the party in yielding a larger overall majority. In the limiting case, \( \gamma \to \infty \), incentives are perfectly aligned which mimics a unitary mapper and our solution converges on the traditional, unitary actor, crack-and-pack results of Gul-Pessendorfer. Analytically, this can be seen by noting that as \( \gamma \to \infty \), the two first order conditions become identical.

In the other limit, \( \gamma \to 0 \), sitting legislators are entirely self-serving. Utility gains occur only when friendly voters are reallocated from opposition districts by deeper packing and thus there is no motive to attempt to flip a district. Hence there is a discontinuity in the manner in which competitive districts (\( p_i \approx 0.5 \)) are treated depending on whether they are currently part of the caucus (\( p_i > 0 \)) or not (\( p_i > 0 \)) (Figure 3). But when gamma is large, sitting legislators are willing to sacrifice their own chances of reelection to increase the expected party seat-share. As a result, not only are competitive majority party seats receiving support, but some of the opposition-held seats near the margin are targeted to be flipped and receive an influx of friendly voters (Figure 4). Thus the discontinuity occurs at a very different place: between those seats that are being cracked—including some currently held by the opposition—and those that are being packed.

These simulations illustrate two other points from the model that are intuitive but not obvious. Notice that cracked districts are not perfectly equalized. Sitting legislators still defend their endowment of friendly voters. When \( \gamma = 0 \), every sitting legislator fights for an equal share of the friendly voters freed up by further packing (hence the horizontal slope of the right side of Figure 3). When \( \gamma > 0 \),
individual legislators recognize that friendly voters contribute more to expected seat share if they are directed toward more competitive districts and this leads to a downward slope. But even when gamma is sufficiently large to enable a caucus to attempt to expand, the cracked districts are not perfectly equalized: the slope of $\Delta p_i$ vs. $p_i$ is flatter than $-1$ thus $p_i + \Delta p_i$ is not equated across districts. This is another improvement on existing, history-free crack and pack results which predict equality across all cracked districts.

Second, notice that treatment of the districts held by the opposition, those to the left of $p_i = 0$, varies according to gamma. Every district that is set to be packed has the smallest geographically possible number for $\Delta p_i$ as friendly voters are transferred out. But the districts targeted for flipping will have positive values. This is important to our empirical work. Critically, this break-point between crack and pack depends on gamma. Presumably, pooling chambers will result in a mixture of gammas. Thus in some cases there will be many targets on the left side of the line and in other cases relatively few. Moreover, because of varying geographic possibilities for redistricting and incumbent quality, those targets may not be neatly ordered by $p_i$ and thus in practice the left side of the graph is likely to be a jumble.
Figure 6. Simulated $\Delta p_i$-s for a 21 chamber district where the red party enjoys an 11-10 majority and control of the map.

Note: When legislators are group-minded, they do seek to flip opposition districts. In some cases, the safest incumbents will even be willing to outright lose voters. The boundary between those opposition districts which are targeted and those which are packed is situation-dependent.

V. Data Sources and Preparation

To test our propositions, we compile historical voting records of precincts across the United States from the Stanford Election Atlas (Rodden and Ansolabehere). The records indicate how individuals within precincts voted over the period of 2004 to 2008 for state gubernatorial, attorney general, secretary of state, controller, treasurer, insurance commissioner, congress, assembly and senate elections. In addition to state level elections, voting records for the presidential election in 2008 are also included. Normal Democratic vote share ($NDV$) is estimated for a precinct by averaging Democratic vote share across all of the aforementioned elections.

Each precinct’s voter data is linked to a shapefile, a geospatial vector data format for geographic information systems (GIS). Along with our precinct voting level GIS data, we extract shapefiles for the state legislative district lines in 2006 and 2015 from the U.S. Census TigerLines database for the lower and upper chambers of each state. The re-drawing of district lines is typically conducted after each census to account for changes in population estimates. As a result, our
2006 state legislative district lines represent districts over the period of 2001 to 2010 and our 2015 state legislative district lines represent districts over the period of 2011 to 2020. Unfortunately, prior redistricting waves are not available as GIS shapefiles so we are limited to these two waves. Fortunately, we nonetheless have a pair of maps from which to calculate changes in partisan vote shares of districts as the map is redrawn.

We combine our precinct level NDV data with our upper and lower chamber state legislative district lines by first converting the geospatial projections of our data into a common coordinate reference system (CRS) through Environmental Systems Research Institute’s (ESRI) ArcMap software. Once projected into a common CRS, we use ESRI’s intersect tool to find the shared area between our precinct level data and the respective upper and lower chamber state legislative district lines for 2006 and 2015. More precisely, we find the percentage of the area in square kilometers of each precinct that falls within a district, to assign precincts to districts.

District level NDV is then computed as the population and area weighted NDV of each of the assigned precincts. For example, if precinct i has an NDV of 0.6 with 1,000 voters and is geographically split 50 : 50 between districts j and k, district j will receive 500 voters with an NDV of 0.6 and district k will receive 500 voters with an NDV of 0.6. As a result of precincts being our smallest measurement unit, we assume a homogeneous geographical distribution of voters within each precinct. District level NDV is computed in this manner for each of the districts in our lower and upper chamber state legislative district lines for 2006 and 2015. To account for potential changes in district names across the redistricting wave in 2010, we compare how the lower (upper) chamber district lines change by analyzing the population weighted area overlap between lower (upper) chamber lines in 2006 and 2015. We intersect the lower (upper) chamber 2006 and 2015 state legislative district lines to find the shared common area between each district in 2006 with each district in 2015. The common areas are then weighted by the share of the original population in each district to estimate the percentage of voters passed on from an original district i to each of the 2015 districts within the same state as district i. We can thus identify which offspring district k corresponds to which parent district i. The offspring is that which takes on the largest fraction of voters originally in the parent.

VI. Empirical Work

Our sample consists of 25 of the 28 states in which a single party controlled redistricting during the 2010 wave (18 Republican, 7 Democrat). Data on control of redistricting are from Justin Levitt’s website All About Redistricting. Alaska, Massachusetts, and Nebraska were dropped from our sample due to an inability to match districts across waves or lack of data on partisan identities of incumbents.
Using our previously described data on normal democratic vote (NDV), we calculate the change in NDV between the parent and offspring. We then adjust the sign so that in each case, a positive value means the district is becoming more favorable for the party in control of redistricting. This $\Delta NDV_i$, a measure of the extent to which the district is made more or less favorable to the party in control, serves as our dependent variable.

Our theory implies that districts currently under the control of the party conducting the redistricting will be treated differently than similarly competitive districts currently in opposition because the candidates likely to run for the majority party in the out-districts are not represented in the bargaining process. We look for evidence of this hypothesis using first a cross-sectional regression that allows for both intercepts and slopes to vary by pre-redistricting control and then a regression discontinuity design. Both analyses exhibit similar results. In each case, we focus on those districts that are within shouting distance of contestation (electoral margin < 0.25).

Figure 5 shows the plot of the binned data with a fitted 2nd order polynomial. The three most important results are all clear from this figure: the discontinuity showing extra support for districts already under control, the downward slope on the right-hand side indicating that help for sitting co-partisans depends on competitiveness of the district, and the much greater variance on the left-hand side which is consistent with our prediction of shifting boundaries between those districts to be packed and those to be targeted for flipping.

The regression discontinuity is estimated using Calonico et al’s `rdrobust` command in Stata with defaults for bandwidth and local polynomial order (1). We show in Table 1 that the results are robust to these decisions and to the sample restriction. The results suggest that, depending on the specification, a marginal district will get between 1.9 and 2.6 percentage points more help if it is represented by a sitting legislator.

Column 1 of Table 2 shows results for the OLS regression counterpart. The constant term shows that the marginal opposition-held seat gets zero help on average. Meanwhile, the marginal seat held by the party drawing the lines gets 2.2 percentage points of help on average, a magnitude closely in line with the regression discontinuity results. We also see the expected relationship whereby the extent of assistance to own-party incumbents declines as the seat becomes safer. The estimated coefficients suggest that seats with margins of victory above 24% receive no further assistance. The surprise from Figure 5 is confirmed as we see no evidence that action towards opposition held seats is systematically related to the margin of recent defeat. The R-squared is likewise an astonishingly low 1 to 3%, on which we will comment more later.

We then split the sample into chambers that are competitive and those that are not, defining competitive as neither party has a 2/3 super-majority. In our sample of chambers whose redistricting is under unified partisan control, 13/25 upper chambers and 15/25 lower chambers are competitive by this definition, giving us
Note: The discontinuity between held and targeted seats is evident. Seats held by the majority (RHS) receive greater infusions of friendly voters the more competitive was the most recent election. Note also that margin of victory explains partisan shifts better for districts already held by the majority.

Figure 7. RD Plot: Preserving takes precedence over flipping

a roughly equal split of districts and sample size across these sub-samples. We find that the result in question—the discontinuity according to whether the seat is currently held—is twice as large in the competitive chambers as it is in the uncompetitive chambers. Again, this is precisely as predicted by our bargaining theory (Proposition 2). In uncompetitive chambers, the value of the public good is lesser (large majorities ensure favorable policy is not dependent on adding an additional seat) thus there is greater emphasis on individual incentives.

Noting that chamber size, political experience, and term of office might affect the bargaining process, our second split is between lower and upper chambers. Lower chambers are, on average, three times larger than their upper chamber counterparts. They are also far more likely to have 2 year terms instead of 4 year terms: 44 of the 49 lower chambers have 2 year terms whereas 38 of the 50 upper chambers have 4 year terms. Nonetheless, we find no significant difference between lower and upper chambers in the magnitude of the effect.

The miniscule values of R-squared, never topping 3% in any specification, sug-
Table 1—Regression Discontinuity Results

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<tr>
<td>CHAMBERS All</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>RD_Estimate</td>
<td>0.0242</td>
<td>0.0236</td>
<td>0.0255</td>
<td>0.0206</td>
<td>0.0194</td>
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<td>Observations</td>
<td>1895</td>
<td>1646</td>
<td>1895</td>
<td>1895</td>
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<td>BW Type</td>
<td>mserd</td>
<td>mserd</td>
<td>mserd</td>
<td>Manual</td>
<td>Manual</td>
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<tr>
<td>Conventional Std. Error</td>
<td>0.009</td>
<td>0.010</td>
<td>0.010</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>Conventional p-value</td>
<td>0.011</td>
<td>0.014</td>
<td>0.011</td>
<td>0.003</td>
<td>0.001</td>
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<tr>
<td>Robust p-value</td>
<td>0.020</td>
<td>0.032</td>
<td>0.017</td>
<td>0.012</td>
<td>0.009</td>
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<tr>
<td>Order Loc. Poly. (p)</td>
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<td>1.000</td>
<td>2.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Order Bias (q)</td>
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<td>2.000</td>
<td>3.000</td>
<td>2.000</td>
<td>2.000</td>
</tr>
<tr>
<td>BW Loc. Poly. (h)</td>
<td>0.055</td>
<td>0.054</td>
<td>0.108</td>
<td>0.100</td>
<td>0.150</td>
</tr>
<tr>
<td>BW Bias (b)</td>
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<td>0.085</td>
<td>0.157</td>
<td>0.100</td>
<td>0.150</td>
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<tr>
<td>Abs(electoral margin) ≤</td>
<td>0.25</td>
<td>0.20</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
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</table>

Note: This table reports the results of five separate specifications over which three parameters are varied: the order of the local polynomial (p), the bandwidth of the local polynomial (h), and the extent to which the sample is limited to competitive districts (absolute value of electoral margin). All of these specifications are significant at conventional levels and these estimation choices have relatively little effect on the magnitude of the effect.

Gest that the shift in the partisan composition of seats is largely unexplained by the sort of maximization that dominates the classic crack-and-pack theories. We have argued that this is consistent with a bargaining model in which a large set of incumbent politicians have the power to threaten reversion to the status quo thereby ensuring that sacrifices for the common good of increased seat share are rare. As a result, the current map is somewhat sticky and maps display path dependence which results in deviation from the unitary-actor optimum. One alternate explanation is that legal constraints constitute an important source of friction inhibiting the optimal map, especially the traditional redistricting principles highlighted by Winburn (2007).

To address this possibility, we calculate measures of population-weighted overlap to determine the fraction of its original voters that a district retained during the redistricting process. We find that districts held by the party in control of redistricting retain a significantly larger fraction of seats (Table 3, first row of coefficients). The average overlap between parent and offspring is 73.4% in our sample. This, along with the low R-squared in Table 2, is strong evidence that whatever constrains shifts in partisan alignment is not a legal constraint on moving district boundaries.
Table 2—OLS Results

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔNDV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chambers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>ΔNDV</td>
<td>ΔNDV</td>
<td>ΔNDV</td>
<td>ΔNDV</td>
<td>ΔNDV</td>
</tr>
<tr>
<td>Own Seat (held by party drawing the lines)</td>
<td>0.0218***</td>
<td>0.0131**</td>
<td>0.0285***</td>
<td>0.0254***</td>
<td>0.0214***</td>
</tr>
<tr>
<td>Margin of victory (defeat for party drawing the lines)</td>
<td>0.0733**</td>
<td>0.0241</td>
<td>0.0985**</td>
<td>0.118*</td>
<td>0.0587*</td>
</tr>
<tr>
<td>Own seat * Margin of Victory</td>
<td>-0.164***</td>
<td>-0.0548</td>
<td>-0.241***</td>
<td>-0.256***</td>
<td>-0.136***</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td>-0.000614</td>
<td>0.00206</td>
<td>-0.00191</td>
<td>-0.00525</td>
<td>0.000756</td>
</tr>
<tr>
<td>Observations</td>
<td>1,918</td>
<td>852</td>
<td>1,066</td>
<td>538</td>
<td>1,380</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.014</td>
<td>0.008</td>
<td>0.023</td>
<td>0.027</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Note: Simple OLS regressions are in line with the RD results reported in Table 1 and highlight three additional results. First, the slopes with respect to electoral margin are significant and of opposite sign indicating, as expected, that friendly voters are transferred from uncompetitive districts to competitive districts. Second, the result is confined to competitive districts. Third, there is no difference between upper and lower chambers despite the systematic differences in term length and chamber size which might have affected bargaining.

VII. Discussion

We have argued that existing theories of partisan gerrymandering are likely to overestimate the degree to which the mapmaker can pursue seat share maximization when drawing the new map. The fixed set of friendly voters is a scarce resource over which members of the majority party have only partially aligned preferences. These voters simultaneously provide the public good of expected seat share and the rivalrous private good of one’s own chance of reelection. Thus we propose to replace the unitary decision-maker with a bargaining framework. This shift delivers two important impediments to the maximization of expected seat share. The first is the emphasis on those districts whose representatives are an active part of the bargaining process. The second is the sense that current representatives have some form of property rights over their current districts and must agree to trade them away. As a result, the existing map becomes an important
point of departure. This introduces a role for history and inertia in the pursuit of seat-share maximization.

But our presumed threat point is a strong requirement: the Nash result that every individual legislator must receive surplus beyond the disagreement point seemingly suggests an underlying structure in which any individual legislator can veto a map. Why, when moving away from the concentration of power in an individual, should we move to the other end of the spectrum? It is likely that the actual process of drawing the lines is done by a small committee in extensive discussion with the broader membership, representing at least some concentration of power. Is it not out of the question that some members should be called upon to sacrifice? Does not the party have the fungible resources and longevity to enable trade credit for such sacrifices?

Perhaps one ought to view our model as representing the other end of a spectrum and thus usefully illustrating the effects of moving some distance in that direction. And yet our empirical results suggest that only a tiny fraction of the shifting of voters produces a net change in the partisan balance of districts in the pattern that would imply improvements in expected seat share. This might be evidence of the difficulties in effecting net changes in vote share that result from the fact that most precincts are mixed and contiguity prevents grabbing distant voters who might offer the needed concentration. We investigate these geographic constraints elsewhere. We believe it is also evidence of the inertia that comes from decentralized bargaining with broadly distributed power to revert to the status quo.

One puzzle that remains is why, among opposition districts, the change in the partisan vote share is not a clear function of the competitiveness of the district. The RD plot (Figure 5) shows the opposite relationship—districts narrowly lost receiving less help than those convincingly lost. Meanwhile, the OLS results, restore the expected sign. One possibility for this confusion has already been discussed: the pooling of chambers pools differing cutoffs. Another possibility for the asymmetry is that the bargaining process requires defending all majority-party districts but that opposition districts are picked off solely based on opportunity. The lack of a relationship could then be the product of either of two branches. Either opportunities are not meaningfully correlated with the competitiveness of the district or the flipping of opposition districts is not pursued by the transference of friendly voters, possibly because that requires those voters be donated from a district already held.

Opportunity might arrive in the form of retirements, scandals, and the possibility of pairing opposition incumbents, none of which would appear in our data. Scandals are likely equally distributed across districts, and would thus simply be noise obscuring any existing relationship, but not fundamentally explaining the lack thereof. Opportunities to pair incumbents would seem to be more useful if the district is actually winnable demographically; there is no partisan gain in pairing two incumbents deep in opposition territory. Retirements may be more
likely in close districts. All of this contributes to the greater variance in the treatment of opposition districts that is evident in the RD plot.

In sum, the net shifts in the partisan composition of voters in state legislative chambers in the 2010 wave are remarkably muted when compared to the predictions of models based on unitary mappers. We suggest this could be explained as the result of a bargaining process in which sitting legislators of the majority party enjoy broadly dispersed power to default to the existing map. Our decentralized bargaining theory further predicts a discontinuity in the treatment of competitive districts already held by the majority party and those currently held by the opposition. Moreover, this discontinuity ought to be greater in chambers where the majority is large. We find support for both of these hypotheses. We thus have specific empirical support for our theoretical proposition that redistricting is a bargaining process that privileges current members of the chamber.

VIII. References

REFERENCES


APPENDIX

In the text, we have described the process of matching parent and offspring districts from successive waves. Ideally, this mapping would be one-to-one and
onto. Unfortunately, there is no single, obvious method by which to produce a mapping that is one-to-one and onto and yet some choices in this mapping method affect the outcome. Nonetheless, we believe we have the proper mapping and believe our results are robust to alternative appropriate mappings and thus relegate this more detailed explanation to the appendix.

Our mapping procedure was this: for every parent district from the prior wave, assign as its offspring district that district from the successor wave to which it (the parent) donated the largest number of voters. Thus if district A were split across districts A’, B’, C’ 20-45-35, B’ would be designated as the offspring of A. To calculate the change in NDV, we would subtract the NDV of A from that of B’. For the regressions, we would be pairing this ∆NDV with the most recent electoral margin in parent district A.

One can imagine several other mapping procedures. The simplest change would be to match parents to offspring rather than the other way around. That is, for every offspring, assign to it as parent that district from which the greatest fraction of the offspring is derived. In the overwhelming majority of cases, this change makes no difference. Consider Table A1 illustrating a hypothetical set of three districts. In this case, the (parent, offspring) pairs are (A,B’), (B,A’), (C, C’) no matter which direction is chosen for the matching. But if we consider the slightly modified example in Table A2, we now see that the direction of the matching matters.

<table>
<thead>
<tr>
<th>% of parent in offspring</th>
<th>Offspring</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Parent</td>
<td>A’</td>
<td>B’</td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>45</td>
</tr>
<tr>
<td>B</td>
<td>75</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>52</td>
<td>48</td>
</tr>
</tbody>
</table>

Matching offspring to parents produces (A,B’), (B, A’), (C, B’) whereas matching parents to offspring produces (A’, B), (B’, C), (C’, C). This also shows how

\footnote{Notice that while the rows must sum to 100%, the columns do not as a result of differential population growth rates. For example, if district X grows much more slowly than the rest of the state, then 100% of district X would be insufficient to furnish the full population of successor district X’ which would need some fraction, say 10%, of district Y in which case the X’ column would sum to 110%.
the matching is neither one-to-one nor onto. In the second case, offspring B’ happens to be the largest recipient from both A and C. Likewise, parent C is the largest donor to both B’ and C’. There are essentially three ways of dealing with this. The first is to accept the match as is. The second is to remove multiple matches according to some priority and rematch the leftover parents and offspring according to some alternate rule. The third is to remove the multiple matches without rematching. None are ideal.

The first method results in a partially complete map in that either all the parents or all the offspring are used but not both. The strength of this approach is that a clear and consistent relationship between parent and offspring is maintained. The third method similarly maintains a clear relationship between the parent and offspring of the maintained matches, with the added benefit of avoiding double-use of any parents or offspring, but at the cost of an incomplete map and a choice over how to prioritize among multiple matches.

Achieving a one-to-one and onto mapping requires the second method, which necessitates another method of matching. Unfortunately, in most cases, one is left matching parents and offspring that have zero overlap. To understand why, consider the five-district example in Table A3. The assignment of offspring to parents results in both A and C wishing to claim B’. As C clearly has greater claim to B’ than does, having donated 70% rather than 45%, we assign B’ to C and search for a new match for A. Unfortunately, both A’ and D’ have already been assigned to B and D respectively. The unassigned offspring is C’, with which A shares no overlap. In this particular instance, one might argue that if we were to assign the contested offspring B’ to A, then C’ could be assigned to C, thus ensuring the secondary pairing also enjoys non-zero overlap. We have experimented with such schemes and found they solve relatively few cases and at the cost of significantly reducing the overlap of the first match.

Our model considers the effect of incumbent preferences on the district in which they run. As they must be a resident to run in the district, a new district with

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12Unlike Congressional redistricting, we almost never have to deal with the loss or gain of a seat during the 2010 wave. The exception is the NY Senate, which added a 63rd seat. In that case, we chose to allow one of the offspring to remain unmatched.
no overlap is not an incumbent-relevant offspring. As such, we wouldn’t expect our theory to be relevant to such matches. Indeed, our core results go through with either offspring-to-parent or parent-to-offspring matching and either allowing many-to-one matches or keeping only the strongest such match. However, when we attempt to rematch the remaining parents and offspring, the resulting noise overwhelms the result. Thus despite the desirability of a one-to-one and onto mapping, the lack of a clear relationship between the rematched parents and offspring makes it clear these districts ought to be left out of the analysis.