HW1

January 23, 2012

1. For each of the following equations, state the order and whether it is nolinear, linear inhomogeous, or linear homogeneous; provide reasons.
(a) $u_t - u_{xx} + 1 = 0$
(b) $u_t - u_{xxt} + uu_x = 0$
(c) $u_t - u_{xx} + xu = 0$
2. Classify each of the equations
(a) $u_{xx} - 5u_{xy} = 0$
(b) $4u_{xx} - 12u_{xy} + 9u_{yy} + u_y = 0$
(c) $4u_{xx} + 6u_{xy} + 9u_{yy} = 0$
3. Consider the general form of second-order linear PDEs

$$Au_{xx} + Bu_{xt} + Cu_{tt} + Du_x + Eu_t + Fu + G = 0$$

under the coordinates transformation

$$\xi = \xi(x,t), \quad \eta = \eta(x,t).$$

Rewrite the equation into

$$A'u_{\xi\xi} + B'u_{\xi\eta} + C'u_{\eta\eta} + D'u_\xi + E'u_\eta + F'u + G' = 0$$

by computing all the second order and first order derivatives in terms of $\xi$and η. (Hint : For example, $u_x = u_\xi \xi_x + u_\eta \eta_x$.) Show that

$$B'^2 - 4A'C' = J^2(B^2 - 4AC)$$

where $J = (\xi_x \eta_t - \xi_t \eta_x)$. When $J \neq 0$, it is a one-to-one mapping, the classfication of PDEs is indepedent of the coordinate system we choose to represent it.