Problem Set 6 – Answer Key

1 – Skippy has a trust fund that pays him $1000/month, which he can supplement by working and earning wages of $10/hr. Skippy’s preference over goods and leisure are captured by the Cobb-Douglas utility function \( u(q_c, \ell) = q_c^{0.6} \ell^{0.4} \). Skippy has 400 hours a month that he can use for leisure or work.

Suppose the government decides to raise revenue by taxing his labor market income at 25%. Skippy will increase his labor supply in reaction to the tax. True, False, or do we need more information? (Explain why you give the answer that you do)

Answer: False. Given his Cobb-Douglas utility function, we know that before the tax, Skippy’s demand for leisure will be given by

\[ \ell = 0.4 \times \frac{1000 + 10 \times 400}{10} = 200, \]
which means he will work \( L = (400 - 200) = 200 \) hrs.

After the tax, Skippy’s demand for leisure will be given by

\[ \ell = 0.4 \times \frac{1000 + (1-0.25) \times 10 \times 400}{(1-0.25) \times 10} = 213, \]
which means he will work \( L = (400 - 213) = 187 \) hrs.

So, given the Cobb-Douglas assumption regarding preferences, Skippy will decrease his labor supply in reaction to the tax.

2 – Consider the labor supply curve depicted below.

![Labor Supply Curve](image)

(a) Sketch a carefully labeled indifference curve/budget constraint graph that would lead to such a labor supply graph.
(b) Suppose everyone in the economy had a labor supply curve similar to the one depicted above. If the tax on wage earnings was increased, what will this do to the amount of labor supplied to the labor market? Describe your answer in detail.

Answer: It is ambiguous. Such a tax increase would effectively shift the labor supply curve in the manner depicted below (bold line). For example, at a market wage of $10/hr, individuals would now supply what they used to supply at $10-t/hr, where t is the amount of the increase in the tax (as now at a market wage of $10/hr they are only taking home $10-t). This would mean at market wages below $20/hr, individuals would work less than they used to meaning the demand curve would be shifted back.

Alternatively, at a market wage of $25/hr, individuals would now supply what they used to supply at $25-t/hr, where t is the amount of the increase in the tax (as now at a market wage of $25/hr they are only taking home $25-t). This would mean at market wages above $20/hr, individuals would work more than they used to, so the demand curve would be shifted out.

So the net effect on labor supply would depend on what fraction of the population makes more than $20/hr versus less than $20/hr.
Suppose that Skippy can no longer work in the labor market due to a freak accident involving a keyboard, two pencils, and florescent lighting. However, he knows he will receive $10,000 next year from his disability claim, but nothing for this year. The going risk-free nominal rate of interest on borrowing and saving is 0.10 and the expected rate of inflation is zero. As a form of worker’s comp for his injury, Skippy’s old employer gives him a choice between a one time payment of $500 this year, or access to a subsidized risk-free nominal interest rate for borrowing of 0.02.

From the above information, we know Skippy will choose the payment. True, False, or can’t we tell from the information given? Please explain and/or show why you give the answer that you do. (Hint: draw graphs of the Budget Set available under each).

**Answer:** Assuming Skippy is not allowed to take a 2% loan and invest it at 0.10, we can’t tell from the information given. To see why, first note that with the $500 payment and the non-subsidized interest rate of 0.10, the most Skippy could consume this year is the $500 payment plus the maximum amount he could borrow given his future payment of $10,000, or 500 + 10,000/1.10 = 9590.91. Alternatively, he could save the $500 payment at an interest rate of 0.10, allowing him to consume up to (1.10)*500 + 10,000 = $10,550 next year. This gives the two intercepts of his budget constraint. The slope is equal to -(1+r) or -1.10. So with the $500 payment, Skippy’s budget constraint is the thick line in the graphs below.

On the other hand, having a subsidized interest rate of 0.02 will not change the maximum Skippy could consume next year, which will simply be $10,000. However, at the lower interest rate, Skippy could borrow up to $10,000/1.02 = $9803.92 to consume this year. This gives the two intercepts of Skippy’s budget constraint under this plan. Moreover, the slope under this plan will be -1.02. So with the subsidized interest rate, Skippy’s budget constraint is the thin line in the graphs below.
Given that budget set under each option has some bundles that are not available under the other budget set, the above graphs show that without more information on Skippy’s preferences, we cannot tell which one he will prefer.

4 – Edgar’s preferences over consumption today versus consumption one year from now are captured by the utility function \( U(c_1, c_2) = c_1^{0.6}c_2^{0.4} \).

(a) If Edgar is endowed with a payment stream of $100 per year and the going real interest rate is 0.10, will Edgar be a borrower, a lender, or neither?
Answer: Working in present value terms, Edgar’s budget constraint is
\[ c_1 + c_2(1/1.10) \leq 100 + 100(1/1.10) \]

Therefore, given his Cobb-Douglas utility function, his demand for consumption today and consumption next year are given by
\[
\begin{align*}
c_1 &= 0.6[100 + 100(1/1.10)]/1 = 114.55 \\
c_2 &= 0.4[100 + 100/(1.10)]/[1/1.10] = 84
\end{align*}
\]

This means Edgar is currently a borrower, as his consumption today exceeds what he is endowed with in the present (i.e. $100).

Note, we can also do this all in future value terms, in which case his budget constraint would be written
\[ c_1(1.10) + c_2 \leq 100(1.10) + 100 \]

and his demand for consumption today and consumption next year are given by
\[
\begin{align*}
c_1 &= 0.6[100(1.10) + 100]/1.10 = 114.55 \\
c_2 &= 0.4[100(1.10) + 100]/1 = 84
\end{align*}
\]

(b) Sketch your answer to (a) on a budget set/indifference curve graph (label carefully).

(c) At what interest rate will Edgar switch his behavior (i.e. how high will the interest rate have to climb for Edgar to become a lender if he is currently a borrower, or how low will the interest rate have to fall for Edgar to become a borrower if he is currently a lender)?

Answer: Since Edgar is currently a borrower, to become a lender he will need an interest rate \( r \) such that the following equation holds:
0.60[100 + 100/(1+r)]/1 < 100

Solving this we get:

r > 0.50