



**CLAREMONT CENTER  
for MATHEMATICAL SCIENCES**

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**THE SECRET LIVES OF POLYNOMIAL IDENTITIES**

by

**Bruce Reznick**

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**Abstract:** Polynomial identities can reflect deeper mathematical phenomena. In this talk, I will discuss some of the stories behind the following four identities (and their relatives). The stories involve algebra, analysis, number theory, combinatorics, geometry and numerical analysis. Fourteenth powers of polynomials will show up.

$$1024x^{10} + 1024y^{10} + (x + \sqrt{3}y)^{10} + (x - \sqrt{3}y)^{10} + (\sqrt{3}x + y)^{10} + (\sqrt{3}x - y)^{10} = 1512(x^2 + y^2)^5, \quad (1)$$

$$x^3 + y^3 = \left( \frac{x(x^3 + 2y^3)}{x^3 - y^3} \right)^3 + \left( \frac{y(y^3 + 2x^3)}{y^3 - x^3} \right)^3, \quad (2)$$

$$(x^2 + \sqrt{2}xy - y^2)^5 + (ix^2 - \sqrt{2}xy + iy^2)^5 + (-x^2 + \sqrt{2}xy + y^2)^5 + (-ix^2 - \sqrt{2}xy - iy^2)^5 = 0, \quad (3)$$

$$\sum_{1 \leq i < j \leq 4} ((x_i + x_j)^4 + (x_i - x_j)^4) = 6(x_1^2 + x_2^2 + x_3^2 + x_4^2)^2. \quad (4)$$

Equation (1) has roots in 19th century mathematics; (2) is due to Viète (1590's); (3) was independently found by Desboves (1880) and Elkies (1995); (4) was used by Liouville (1859) to prove that every positive integer is a sum of at most 53 4th powers of integers.

**About the speaker:** Bruce Reznick received his B.S. at Caltech (1973) and Ph.D. at Stanford (1976), both in mathematics. He joined the faculty at the University of Illinois in Urbana-Champaign in 1979, where he is now a Professor of Mathematics. He was a member of Caltech's 1st place Putnam team in 1971 and 1972 and was on the Putnam problems committee in the early 1980's. He has had a Sloan Fellowship and is currently an Associate at the UIUC Center for Advanced Study. In 2009, he received Undergraduate Teaching Awards both from his College and from the Campus. He has written more than 60 research publications as well as "Chalking It Up: Advice to a New TA". He has advised six completed and four current Ph.D. students. His research interests focus on combinatorial methods in algebra, analysis, number theory and geometry, often involving polynomials, polytopes and integer sequences.

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**Wednesday, September 21, 2011, at 4:15pm**

Millikan Auditorium, Pomona College

Refreshments at 3:45 p.m. in Millikan Auditorium & wine and cheese after the talk in Harry's Room  
(Millikan 209)

*The dinner will be hosted by Prof. Lenny Fukshansky.*

*Please contact Prof. Fukshansky if you are interested in attending the dinner.*