REBMI

RESEARCH EXPERIENCES at the BIOLOGY-MATHEMATICS INTERFACE

and

CLAREMONT CENTER for MATHEMATICAL SCIENCES

Student Poster Session

Research in the Mathematical Sciences

Summer 2011

Wednesday, September 14, 2011

4:15 - 5:15 pm

Marian Miner Cook Athenaeum Courtyard

Claremont McKenna College

ABSTRACTS

REBMI–1

Rate Constant Wizard$^{\text{TM}}$: A New Tool for Determining Kinetic Binding Rates from Quartz Crystal Microbalance Data.

Liz Sarapata (HMC), Theodore Zwang (POM), Gabriella Heller (POM), Ami Radunskaya (POM) and Matthew Sazinsky (POM). *Funded by the National Science Foundation. (NSF–0634592).*

The Quartz Crystal Microbalance (QCM), a tool used by chemists, can detect small amounts of mass attaching to a binding site with a high degree of precision. This quality makes the QCM ideal for determining the binding rates associated with various chemical reactions. Despite this potential, few equations associating raw QCM data with the binding rates have been developed, and no automated tools using the existing equations have been released to the public. We present two different equations for determining rates of binding from QCM data, one established in literature and one developed by our team. The new equation accounts for the binding of nonspecific mass and presents strategies to isolate only the specific mass given by a described chemical reaction. Both equations were tested and automated to be included in an open-source automated tool for chemists, Rate Constant Wizard.$^{\text{TM}}$

REBMI–2

In Vitro Control and Actuation of a Biological Lever System: The Effects of Passive Elements in the Joint on the Mechanical Output of the Limb.

Rebecca Salzman (CMC) and Jon Schwartz (HMC). Advisor: Anna Ahn (HMC). *Funded by the National Science Foundation. (NSF–0634592).*

The passive elements of the intact ankle joint of adult American bullfrogs (Rana catesbeiana) were investigated by determining the position output after stimulation while the muscle-tendon units were both intact and severed. The unstimulated, antagonist muscle-tendon unit contributes in determining the rest position of the joint. However, after the muscle is stimulated, the antagonist muscle-tendon unit has no effect on the position output and recoil of the limb, thus suggesting the large effect of the nonmuscular passive elements such as the ligaments in frog ankle joints.

REBMI–3

Measurement of Manual Dexterity with State Dependent Perturbations of the Index Finger.

Alison Kent (PIT) and Dana Canfield (SCR). Mentors: Francisco Valero-Cuevas (USC), Alex Reyes (USC), Brandan Holt (USC), Sudarshan Dayanidhi (USC), Manish Kurse (USC) and Jason Kutch (USC). *Funded by the National Science Foundation. (NSF–0634592).*

A Matlab program that sent a perturbation to a subject's index finger when a given force was applied to a spring and maintained a certain level of stability was written. The variation of force and acceleration during a perturbation compared with when the subject was holding the spring steady was examined. A period of instability during and directly following the perturbation and an eventual leveling of force and acceleration that would mirror pre-perturbation states was expected. A peak in acceleration and decline in force directly following a perturbation was observed and soon after, a leveling of both occurred. There was a definite change in acceleration and force when a subject was perturbed and perturbation interfered with subjects' ability to stabilize the spring post-perturbation.

REBMI–4

Dynamical Analysis of Motion in Patients with ACL Injury and Developing Chicken Embryos.

Timothy Law (CMC), Akshata Ramesh (SCR) and Dr. Francisco Valero-Cuevas (USC).
Funded by the National Science Foundation. (NSF–0634592).

Current motion analysis labs utilize only qualitative visual methods to record and analyze motion. Hence, there is a need for a more substantial and quantitative method for motion analysis, to investigate lower-limb motion impairment in athletes. The principal objective of the first study was to test the viability of using five tri-axial accelerometers and a Bluetooth data transmission setup to record and analyze movement in the lower limbs. This set-up would be used to explore possibilities of conducting clinical assessments of lower-limb motion impairment in athletes, as they perform complex cutting and running tasks. This is crucial for determining the extent to which the knee heals following ACL reconstruction surgery, given different recovery periods. Initially, five tri-axial accelerometers attached on both the lower limbs and back, were used to collect data from healthy subjects. This was done in order to determine an efficient method of visualizing the data for analysis. The application of Principal Component Analysis (PCA) yielded an efficient way of visualizing the 15 channels of data that were gathered from the five tri-axial accelerometers. The goal of the second study was to use the model chick embryo to better understand the importance of human fetal movement during development, and to investigate the contribution of light to growth in pre-term infants. Prior research findings have utilized analysis of force, stride length, and EMG readings, but have not shed sufficient light in understanding kinematic movement of chicks during development. Therefore, in order to further analyze chicken embryo locomotion, this study investigates the viability of measuring acceleration of embryo movement within fertilized eggs. Similar to the ACL study, the objective was to determine the best method to attach tri-axial accelerometers onto the eggs, and to identify patterns of movement over time in developing chicks with normal light exposure. With the application of an external analog filter, the minute acceleration signals can be magnified to reveal a more substantiate portrait of chicken embryo locomotion.

REBMI–5

The Effects of Variable Neural Recruitment Strategies on Biomechanics during Walking.

Chris Jerry (CMC), Christian Stevens (HMC), Teri Cinco (HMC) and Parker Martin (HMC). Advisor: Anna Ahn (HMC). *Funded by the National Science Foundation. (NSF–0634592).*

In sedentary subjects, shorter heel length and larger calves correlate with a bias towards recruitment of the medial gastrocnemius (MG) over that of the lateral gastrocnemius (LG). This difference in muscle recruitment suggests a variance in foot locomotion. It has been observed that while walking, subjects that show greater gastrocnemius activity, independent of their walking speed, have higher peak pressures under the medial side of the foot. In this study, we investigate whether neural recruitment patterns affect the distribution of pressure underneath the foot during walking at various speeds. Data collection involved the use of sEMGs to measure muscle activation in addition to Qualisys motion capture system, ultrasound, and pressure sensors to analyze joint angles, muscle size, and plantar pressure, respectively. Consistent with previous studies, 6 of the 13 subjects were MG-biased, while the remaining 7 were unbiased. MG-biased subjects continued to exhibit larger calves and shorter heels as well as exhibiting no significant difference in ankle angle as a percent of the stride. In both MG-biased and unbiased subjects the PTI under the medial side of the foot divided by the sum of the medial and lateral plantar PTIs at all speeds was about 0.66. Medial PTIs were slightly lower in MG-biased as compared to unbiased individuals. : There was no statistically significant correlation between higher PTIs under the medial side of the foot in MG-biased individuals as compared to unbiased individuals. However, at certain speeds there is a correlation between MG-biased individuals and an increase in lateral plantar pressure distribution.

FJ–1

On Integral Well-Rounded Lattices in the Plane.

G. Henshaw (Wesleyan), P. Liao (CMC), M. Prince (HMC), X. Sun (CGU), S. Whitehead (POM). Mentors: L. Fukshansky (CMC). Funded by the Fletcher Jones Fellowship.

We investigate distribution of integral well-rounded lattices in the plane, producing a complete parameterization of the set of their similarity classes by solutions of the family of Pell-type Diophantine equations of the form $x^2 + Dy^2 = z^2$ where $D > 0$ is squarefree. We then apply our results to the study of the greatest minimal norm and the highest signal-to-noise ratio on the set of such lattices with fixed determinant, also estimating cardinality of these sets for each determinant value. This investigation extends previous work of Fukshansky et al. in the specific cases of integer and hexagonal lattices and is motivated by the importance of integral well-rounded lattices for discrete optimization problems. We separately study a special subclass of integral well-rounded lattices which come from ideals in quadratic number fields; these are called *ideal lattices*. We extend recent results of Fukshansky and Petersen on planar well-rounded ideal lattices, giving their complete characterization. This poster presents our joint work with our project advisor L. Fukshansky.

FJ–2

Linear Tests of Uniformity for Data Defined on Polygons and Finite Tori.

Connor Ahlbach (HMC), John Choi (HMC), Laura Passarelli (Scripps), Flora Xu (CGU). Advisor: Michael Orrison (HMC). Funded by the Fletcher Jones Fellowship.

Suppose respondents in a survey are asked to choose an element from a finite set X. If we assume their responses are governed by an underlying probability distribution P, then it is natural to wonder whether P is actually the uniform distribution defined on X. In this poster, we present the results of our study of linear tests of uniformity when X is the set of points on a discretized circle or torus. In particular, we construct several straightforward tests of uniformity, derive formulas for their associated degrees of freedom, and run the tests on two example data sets to demonstrate their usefulness.

FJ–3

Constructing Integrable Systems From Graded Classical r-Matrices.

Keith McHugh (POM), Aaron Pribadi (HMC), Peter Fedak (HMC) and Sundeep Sampath (CGU). Advisor: Gizem Karaali (POM). Funded by the Fletcher Jones Fellowship.

We examine methods of constructing integrable systems from solutions of the graded classical Yang-Baxter equations (CYBE). This process is well-understood in the non-graded case; we extend its scope to Lie superalgebras by following the work of Zhang, Gould, and Bracken (1991). In particular, we explicitly describe an approach to go from the r-matrix solutions of the graded CYBE to an integrable classical system on a supermanifold. We illustrate our method with examples of integrable systems and examine how they relate to their non-graded counterparts.

CMC–1

Enhancements of the Rack Counting Invariant via N-reduced Dynamical Cocycles.

Aparna Sarkar (POM) and Prof. Sam Nelson (CMC). Research funded by CMC.

We introduce the notion of N-reduced dynamical cocycles and use these objects to define enhancements of the rack counting invariant for classical and virtual knots and links. We provide examples to show that the new invariants are not determined by the rack counting invariant, the Jones polynomial or the generalized Alexander polynomial.

HMC-1

Numerical Simulations of a Thin Film with Surfactant.

Cameron Conti (HMC), Eric Autry (HMC), Greg Kronmiller (HMC) and Rachel Levy (HMC). Funded by Harvey Mudd College.

The presence of a surfactant on a thin liquid film, such as the lining of the lungs or the tear film on the eye, creates a surface tension gradient, which causes the film to flow and advect the surfactant across the surface. In this work, a numerical solver developed by J. Claridge, R. Levy, and J. Wong that describes the dynamics of this system is tested and validated. A grid refinement test verifies that the code is first order, and an investigation of the time step needed for a simulation with a given resolution to complete shows that the maximum time step is approximately proportional to the 4.5th power of the cell size. The symmetry of solutions involving initial conditions that are not aligned with the grid is shown to improve as the grid is refined. Physical oscillations that arise from a fourth-order term in the equations have also been observed.

HMC–2

A Turan-type Problem for Circular Arc Graphs.

Rosalie Carlson (HMC) and Kevin O'Neill (HMC). Advisor: Francis Su (HMC). Funded by the National Science Foundation.

A circular arc graph is the intersection graph of a collection of connected arcs on the circle. We consider a Turán-type problem for circular arc graphs: for n arcs, if m and M are the minimum and maximum number of arcs that contain a common point, what is the maximum number of edges the circular arc graph can contain? We establish a sharp bound that, given a fixed minimum m arcs that contain a common point, can be used to show that if the circular arc graph has enough edges, there must be a point that is covered by at least M arcs. In the case $m = 0$, we recover results for interval graphs established by Abbott and Katchalski (1979). We suggest applications to voting situations with interval or circular political spectra.

POM–1

Brouwer Degree in Two Dimensions.

Bob Lutz (POM) and Adolfo Rumbos (POM). Funded by Pomona College.

The Brouwer degree is a concept with origins in topology and applications in many aspects of nonlinear analysis whose value can guarantee solutions to the equation $y = f(x)$ in a suitable space. We construct a new, analytical definition of the Brouwer degree in $\mathbb{R}^2$ by examining the planar case of the Kronecker index, an analogue of the complex winding number. Given a C^1 vector field $F(x,y) := P(x,y)\hat{\imath} + Q(x,y)\hat{\jmath}$ and C^1 Jordan curve ∂D in $\mathbb{R}^2$, we define the Kronecker index of $F = F(x,y)$ around ∂D by

$$\text{index}_K(F, \partial D) = \frac{1}{2\pi} \oint_{\partial D} \frac{PdQ - QdP}{P^2 + Q^2}. \tag{1}$$

This integer–valued expression calculates the number of times the curve $F(\partial D)$ "winds" around the origin and conveniently describes the Brouwer degree for the case in which F and ∂D are C^1. In order to define the degree generally for continuous F and ∂D via the formulation in (1), we must define suitable C^1 approximations of our vector field and curve and justify these choices. Applications will be briefly discussed.

REU–1

Non–Consecutitve Pattern Avoidance in Full Binary Trees.

Mike Dairyko (POM), Samantha Tyner (Augustana College) and Casey Wynn (Hendrix College). Advisor: Lara Pudwell (Valparaiso). Funded by the National Science Foundation.

We consider the enumeration of full binary trees avoiding non-consecutive binary tree patterns. We begin by modifying a known algorithm that counts binary trees avoiding a single consecutive tree pattern. Next, we use our algorithm to prove several theorems about the generating function whose nth coefficient gives the number of n-leaf trees avoiding a pattern. In addition, we investigate and structurally explain the recurrences that arise from these generating functions. Finally, we examine the enumeration of binary trees avoiding multiple tree patterns.

REU–2

Music Transcription with Markov Models.

Esther Gross (UMBC), Elizabeth Schofield (HMC), Lisa Watanabe (UC Berkeley), and Guang Yang (Rice). Advisors: Talithia Williams (HMC) and Gabriel Chandler (POM). Funded by the National Science Foundation.

In the past, both Hidden Markov Models (HMMs) and template-matching have been employed in sound analysis. This project focused on the combination of these two techniques in identifying and transcribing the occurrences of three percussion instruments–bass drum, snare drum, and hi-hat cymbals–over brief drum interludes. The audio recordings of these drum interludes were interpolated and then converted into spectrograms in the time-frequency domain. Then templates of the three instruments were created from the spectrograms, alongside noise templates for both the high and low frequencies. Correlation values from comparing these templates to respective sixteenths of drum breaks then provided observation data for a time-dependent Hidden Markov Model. Using a Gaussian distribution to account for the lack of discrete observation conditions, the HMM generates the most likely sequence of drum hits per sixteenth note for each audio recording via the Viterbi Algorithm. This strategy yielded modest results in simple application, but shows promise if both the Gaussian distribution and the templates are created with more data.