MATH 30 - 2 & 3, SPRING 2018, PRACTICE TEST II

Disclaimer: This practice test DOES NOT serve as an indication of the contents of the actual test. It only suggests a possible format.

Please print your name clearly!

Name:____

Please show all your work, that is explain every step of your solution - it is your work, not the answer, that is being evaluated. When asked to prove a statement, make sure to provide reasoning behind each claim you are making in the process of the proof. The use of calculators or any other electronic devices is prohibited during the test. You are also not allowed to use any study materials except for those provided to you during the test. Cheating is strictly prohibited, and will be prosecuted. Good luck!

Problem 1 (40 points). Compute the following limits or prove they do not exist. Show all your work.

a) - (10 points) $\lim_{x\to-\infty}\left(x+\sqrt{x^2+2x}\right)$

b) - (10 points) $\lim_{x\to 0^+}\tan^{-1}(\ln x)$

c) - (10 points) $\lim_{x\to\infty}(e^{-x}+\sin x)$

d) - (10 points) $\lim_{x\to\infty}\frac{x^6+1}{1+x^2-x^6}$

Problem 2 (30 points). Determine where are the following functions continuous.

a) - (10 points) $f(x) = \frac{x-2}{x^2+2}$.

b) - (10 points) $g(x) = \frac{\sqrt{x}}{\sin(x)}$.

c) - (10 points) $h(x) = \begin{cases} 3x+2 & \text{if } x < 2 \\ x^3 & \text{if } 2 \leq x \leq 3 \\ 3x^2 + x - 1 & \text{if } x > 3 \end{cases}$

Problem 3 (20 points). Let

$$f(x) = \sqrt{x}$$

throughout this problem.

a) - (10 points) Use the limit definition of the derivative to find the function $f'(x)$ and specify its domain.

b) - (10 points) Find an equation for the tangent line to the graph of $y = f(x)$ at $x = 4$.

Problem 4 (20 points). Find derivatives of the following functions.

a) - (10 points) $f(x) = e^x(x^3 - x^2 + x)$

b) - (10 points) $g(x) = \frac{e^x}{x^3 - x^2 + x}$

Problem 5 (10 points). Let

$$f(x) = \frac{\sin x}{x}.$$

This function is continuous everywhere except at $x = 0$. Use the fact that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

to define a function $g(x)$ that is continuous everywhere and is equal to $f(x)$ at all $x \neq 0$. Is this function differentiable at $x = 0$? If so, use the limit definition of the derivative to compute $g'(0)$.

Hint: $\lim_{x \to 0} \frac{\sin x - x}{x^2} = 0$.