MATH 31 - 3, SPRING 2015, PRACTICE TEST II

Disclaimer: This practice test DOES NOT serve as an indication of the contents of the actual test. It only suggests a possible format.

Please print your name clearly!

Name:_____

Please show all your work, that is explain every step of your solution - it is your work, not the answer, that is being evaluated. When asked to prove a statement, make sure to provide reasoning behind each claim you are making in the process of the proof. The use of calculators or any other electronic devices is prohibited during the test. You are also not allowed to use any study materials except for those provided to you during the test. Cheating is strictly prohibited, and will be prosecuted. Good luck!

Problem 1. Find the values of the following definite integrals, if they exist:

a)
$$\int_1^\infty \ln x \, dx$$

b)
$$\int_0^1 \frac{x}{1-x^2} \, dx$$

c)
$$\int_0^\infty \frac{1}{e^x + e^{-x}} \, dx$$

d)
$$\int_0^{\pi/3} \frac{\sec^2 x}{1 - \tan x} \, dx$$

e)
$$\int_0^2 \frac{x^2+2}{x^3-1} \, dx$$

Problem 2. Suppose that the region

$$R = \left\{ (x, y) : x \geq 1,\ 0 \leq y \leq \frac{1}{x} \right\}$$

is rotated around the x-axis. Let S be the resulting solid.

a) Find the volume of S.
b) Find the surface area of S.

Problem 3. A rumor is being spread around the community of population 100. On Monday at 8:00 am, 10 people have heard it, while at 10:00 am already 30 people know the news. Use the logistic growth model to find an explicit expression for $Q(t)$, the number of people who have heard the rumor at time t, as a function of time, measured in hours. At what time will half the population have heard the rumor?

Problem 4. Let $a > 0$ be a real number. Consider an initial value problem

$$y' + \frac{y}{x} - \ln x^a = 0, \quad y(1) = 1, \tag{1}$$

where $x > 0$. Solve (1).

Problem 5. All the curves in this problem are polar.

a) Find the area of the region that lies inside the curve $r = 1 - \sin\theta$ and outside the curve $r = 1$.

b) Find the length of the polar curve $r = 1 + \cos\theta$.