On the Role of Plea Bargaining and the Distribution of Sentences in the Absence of Judicial System Frictions

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Abstract
This paper examines the potential role that plea bargaining may serve in an environment where trials and incarceration are costless, there is no chance of convicting innocent defendants, and coercion is not necessary to obtain further information from arrested defendants. This analysis shows that even in the frictionless judicial system environment outlined above, a risk-averse society may still find it optimal to resolve a large fraction, or even all of its cases through plea bargaining. However, this is not always the case. If defendants are generally risk-loving, mutually acceptable plea bargains may not be possible, but when they are, the plea bargain sentences will necessarily fall short of even the expected sentence from going to trial, regardless of the relative bargaining power between prosecutors and defendants.

Keywords: Plea Bargaining, Sentencing Policy, Prosecutor Behavior.

*Much of this work was done while I was an NICHD Research Fellow in Population Studies at the RAND Corporation.
1 Introduction

One criticism often levied at the American judicial system is that excessive plea bargaining leads to unwanted leniency and disparity in sentencing. It is quite easy to see why this critique arises, as in 2002, over 90 percent of state felony convictions arose through plea bargaining, with the median sentence for those individuals whose cases were adjudicated through guilty pleas being only 30 percent as long as the median sentence received by those who were convicted at trial (Durose and Langan, 2005). Indeed, concerns of leniency and sentencing disparity caused Alaska to ban plea bargaining practices in 1975, and more recently, prompted then Attorney General John Ashcroft to issue a memo to U.S. Federal Prosecutors to significantly curtail their plea bargaining practices (Ashcroft, 2003).

At first glance, these policies may seem justified responses to the sentencing outcomes cited above. However, there exist several arguments for why the apparent leniency and large sentencing variation that accompany widespread plea bargaining should not necessarily be curtailed. One argument is that trials are exceedingly expensive, so society is made better off by using plea bargaining to avoid these costs, even at the expense of often imposing sentences far below that which society deems correct for the crime committed. A related argument is that prosecutors have only a limited amount of time at their disposal and conviction success at trial is an increasing function of time spent preparing. Therefore, if most cases were brought to trial, prosecutors would have little time to spend prosecuting each individual case, leading to a high rate of acquittal. Through extensive plea bargaining, prosecutors can give themselves a better chance of securing convictions for those cases that do still go to trial, while assuring those who plea bargain are still punished (albeit at a less than desired

\footnote{Note, this result generally holds if one also controls for conviction charge.}
A third argument that has been made for why allowing lighter sentencing due to extensive plea bargaining may be optimal is the possibility that some defendants charged by police may be innocent. As discussed by Grossman and Katz (1983), Reinganum (1988), Kobayahsi and Lott (1992), and Baker and Mezzetti (2001), plea bargaining may be used as a way to manage the tension between wanting to punish the guilty and release the innocent, when it is not possible to perfectly distinguish between the two types of defendants. Relatedly, Franzoni (1999) proposes a model of plea bargaining that combines the possibility that plea bargaining can sort (at least partially) the guilty defendants from the innocent defendants, and the fact that plea bargaining can save resources (in this case, resources devoted to investigations prior to trial).

A fourth argument for the potential optimally of lighter sentencing through plea bargaining is that society may find it optimal to let some defendants off easier in exchange for information that leads to the arrest and conviction of other criminals. Indeed, the more restrictive Federal plea bargaining policy initiated by Ashcroft recognizes this role for plea bargaining and makes explicit allowances for the use of such plea bargains.

While all of the above arguments provide legitimate reasons for why lighter sentencing due to plea bargaining may not necessarily be a reason for policy changes, these arguments all rely on plea bargaining being optimal only as a “second-best” response. In other words, the justification for offering relatively short plea bargain sentences to many defendants is not that such a policy is in and of itself optimal, but rather it is the best thing that can be done given that society can not provide prosecutors with infinite resources and/or make the arrest process perfect. The aim of this paper is to examine the potential role plea bargaining may play even in the absence of any of the above judicial system
frictions that have previously been used to justify plea bargaining. Specifically, this analysis examines what will be the optimal use of plea bargaining, as well as the resulting distribution of plea bargain sentences, when resolving a case through trial costs no more money or time than resolving a case through plea bargaining, and the prosecutor is not concerned about wrongly convicting innocent defendants or obtaining information in order to capture other criminals still at large.

As the analysis below makes clear, in the absence of any of the judicial system frictions discussed above, prosecutors may still want to use plea bargaining as a means to mitigate the risk from going to trial—a finding previously highlighted by Grossman and Katz (1983). However, unlike Grossman and Katz (1983), the analysis below shows how the extent to which prosecutors will be able to do so, as well as the distribution of sentences that may result from doing so, will depend substantially on the assumptions made regarding the risk preferences of both prosecutors and defendants.

Specifically, in order to insure against risk, risk-averse prosecutors would be willing to offer a given defendant a plea bargain sentence somewhat shorter than the expected sentence from taking the defendant to trial. Similarly, if defendants are also risk-averse, they would be willing to accept a plea bargain sentence somewhat longer than the expected sentence from going to trial. Therefore, if both prosecutors and defendants are risk-averse (the assumption implicitly maintained by Grossman and Katz (1983)), prosecutors and defendants should always be able to negotiate a mutually acceptable plea bargain. However, the plea bargain sentence for each defendant can be greater than, less than, or equal to the expected sentence from going to trial, depending on the relative degree of risk-aversion of prosecutors versus defendants and the relative bargaining power of each party.
On the other hand, as Polinsky and Shavell (1999) argue, it may be more plausible to assume defendants are actually risk-loving in sentence length. As I show below, this can preclude the possibility of a mutually acceptable plea bargain even when prosecutors are risk-averse. However, when plea bargains can be struck, the plea bargain sentences will necessarily fall short of the expected sentence from going to trial, especially for those defendants with only moderate evidence against them. This can result in sentencing distributions that appear far more lenient than would be implied by the statutory sentence from conviction at trial and the relevant probabilities of conviction at trial.

In general, this analysis shows that concerns about widespread plea bargaining, and/or apparently very lenient sentencing arising from plea bargaining, may be unfounded even when one believes the judicial system to be relatively free of many of the frictions that have previously been used to justify plea bargaining.

### 2 Model

As discussed in the introduction, this model attempts to characterize the use of plea bargaining and the distribution of sentences under plea bargaining. However, as also discussed above, I abstract from the possibility that prosecutors may want to use plea bargaining to manage uncertainty regarding defendant guilt, use as leverage for obtaining information about other criminals, or avoid the costs associated with trials, appeals, and long incarceration terms. Therefore, the model solely focuses on the role of plea bargaining in a frictionless judicial system, meaning that all of the results come out of the assumptions made regarding societal (and therefore prosecutors as agents of society) and defendant preferences.

There are several things to consider regarding prosecutor/societal preferences regarding sentencing criminals. One is from a crime reduction perspective. It
is well documented that from mid-teen years on, individuals become less and less likely to engage in criminal activity as they age (Sampson and Laub, 2003). Therefore, the amount of crime foregone by locking an individual up for an additional year is likely to fall the longer the individual has already been locked up. This means that if society incurs a benefit for each crime averted through incarcerating a particular individual, society would incur positive but diminishing marginal utility in sentence length. Society/prosecutors may also have a direct “taste” for imprisoning a defendant, in the sense they feel they are made better off simply through the act of incarcerating defendants. Once again however, it seems relatively uncontroversial to suppose that the benefit society, and therefore prosecutors, incur from the first year of incarcerating a criminal is greater than from the twentieth year. Indeed, if this was not the case, then if the net benefit to society from incarcerating a given defendant for a year is positive, then society will also see the net benefit to incarcerating this individual for life to be positive. Therefore, if the benefit society incurs from incarcerating defendants does not diminish in sentence length, then society should either want to see convicted defendants receive no prison time or a life sentence—a conclusion few people are likely to agree with.²

Given the above discussion, let prosecutor preferences with respect to the sentence length \( z \) they are able to impose on a defendant be given by a function \( U(z) \), where \( U'(z) > 0 \) and \( U''(z) < 0 \) for all \( z > 0 \), meaning prosecutor utility is an increasing concave function in the sentence length. In words, assume prosecutors become better off the longer the sentence they are able to impose on a defendant, but the marginal utility they incur from imposing a longer sentence is decreasing in the sentence length.³ Note that an important implication of

²Obviously, society/prosecutors may also care about deterring future criminals via sentencing. However, I will delay my discussion of this motivation for sentencing until later in this section.

³Note, such preferences could easily be amended to assume that prosecutor utility decreases in sentence length for sentences beyond some \( \bar{z} \). However, this will not change the
this formulation is that it means prosecutors are risk-averse with respect to sentence length.

For defendants, utility clearly must diminish in sentence length. Therefore, if we let a function $V(z)$ represent defendant preferences over sentence length $z$, it must be the case that $V'(z) < 0$ for all $z > 0$. However, it is not apriori clear what the second derivative of the defendant utility function should be, or in words, whether the marginal disutility of another month of incarceration is increasing or decreasing in the length of time they have already been there. On the one hand, economists generally assume that individuals are risk-averse, preferring a sure thing over a gamble, even if the gamble has a greater expected nominal value. In this context, this would imply that $V''(z) < 0$ for all $z > 0$, or that $V(z)$ is a decreasing but concave function in $z$. Intuitively, this formulation says that the marginal disutility of another month in prison incurred by an individual who has only been in prison a month is less than it is for an individual who has been in prison for 10 years.

One the other hand, the marginal disutility of another month of imprisonment may actually decline over successive years of imprisonment. As argued by Polinsky and Shavell (1999), “disutility of this form might arise because a person becomes accustomed to prison life or because he ceases to care as much about those he knew from the outside.” Relatedly, Polinsky and Shavell (1999) point out that the stigma cost of prison may be unrelated to sentence length, and moreover, the brutalization and humiliation faced by prisoners may be particularly acute early on, but may recede over time as the prisoner ages, gets more experienced with prison behavior, and possibly makes a acquaintances that prevent such actions from continuing (e.g. joins a gang). All of these arguments suggest that the marginal disutility of prison diminishes over time, analysis below as long as this $z$ is greater than or equal to the “statutory” sentence, or the sentence the defendant would receive if convicted at trial (below denoted as $\bar{z}$).
meaning $V''(z) > 0$ for all $z > 0$. A direct implication from this formulation of defendant preferences is that defendants will be risk-loving with respect to sentence length. As discussed in the introduction, this is in contrast to the analysis of Grossman and Katz (1983), who assume that all defendants are risk-averse. The analysis below examines the implications under both formulations of defendant preferences.

Given these defendant preferences, we can also think about a deterrent motivation for prosecutors with respect to sentencing. Under a deterrent motivation, the societal/prosecutor benefit to imposing a longer sentence will be in direct proportion to the disutility such a sentence imposes on a defendant, as the more disutility caused via punishment, the greater the disincentive for engaging in crime. Clearly, under this perspective, the longer the sentence, the greater the utility of society/prosecutors, meaning $U'(z) > 0$. However, the marginal utility of increasing sentence length will depend on the marginal disutility a defendant incurs from a longer sentence. If, as argued above, defendants incur diminishing marginal disutility from incarceration, then like with respect to the other motivations discussed previously, a deterrence motivation will also imply that $U''(z) < 0$, or that societal/prosecutor utility is increasing at a decreasing rate in sentence length. Therefore, deterrence goals are certainly not inconsistent with the assumptions made above.

The final component of the model is to assume that defendants are each characterized by a parameter $p \in (0, 1)$, which gives their probability of being convicted by a jury if their case were to go to trial. This parameter $p$ can be interpreted as the “strength of the case” against a defendant. Assume that this probability of conviction at trial is known to both the prosecutor and the defendant at the time of any plea bargain negotiations. Note that this is consis-

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4This conclusion would also be true if defendants discount their future substantially, so having their freedom taken away next month creates higher disutility than having the same freedom taken away 2 years from now.
tent with the notion of disclosure. Moreover, as can be confirmed below, since
prosecutors are risk-averse, they have no incentive to withhold evidence in order
to make defendants believe their probability of being convicted at trial is lower
than the true \( p \), since prosecutors want to obtain the longest plea bargain possi-
ble. We can also assume that prosecutors cannot credibly manufacture evidence
to make defendants believe their probability of being convicted at trial is any
greater than their true \( p \).

Finally, for simplicity, assume that the sentence that will be imposed on a
defendant if convicted by a jury will be of length \( \tau > 0 \), which will be referred
to as the “statutory sentence.” Alternatively, if the defendant is acquitted, as-
sume he receives a sentence length of zero. Without loss of generality, we can
normalize the utility functions so that \( V(0) = 0 \) and \( U(0) = 0 \).

2.1 Prosecutor Behavior

Given the prosecutor preferences defined above, note that for a defendant with
strength of case \( p \), an expected utility maximizing prosecutor would never be
willing to offer a plea bargain sentence \( z \) such that

\[
U(z) < pU(\tau),
\]

where the term on the right-hand side of the above equation is the prosecutor’s
expected utility from going to trial. Given our assumptions regarding the \( U \)
function, equation (1) implicitly defines a plea bargain sentence length function
\( z^{\text{min}}(p) \), where given a defendant with strength of case \( p \), a prosecutor will
never find it optimal to offer a plea bargain sentence shorter than \( z^{\text{min}}(p) \), with
\( z^{\text{min}}(p) \) equating the right-hand side of equation (1) to the left-hand side for a
given \( p \). Moreover, it can easily be shown that given our assumptions regarding
the \( U \) function, it will be true that \( \frac{\partial z^{\text{min}}(p)}{\partial p} > 0 \) and \( \frac{\partial^2 z^{\text{min}}(p)}{\partial p^2} > 0 \), or that
$z^{\text{min}}(p)$ is an *increasing convex* function of $p$ for all $p \in (0, 1)$, and $z^{\text{min}}(0) = 0$ and $z^{\text{min}}(1) = 1$.\footnote{Formally, from equation (1) we can define a plea bargain sentence length $z^{\text{min}}(p)$ to equal

$$z^{\text{min}}(p) = \gamma(pU(\bar{\pi})),$$

where $\gamma$ is the inverse of the prosecutor’s utility function $U$. Moreover, noting that $\gamma$ is the inverse function of $U$, then we know $\gamma'(z) = \frac{1}{U'(\gamma(z))}$. Since $U'>0$, this implies $\gamma'(z)>0$. Furthermore, using the chain rule, we get $\gamma''(z) = -\frac{1}{U'(\gamma(z))^2}U''(\gamma(z))\gamma'(z)$. Therefore, since $U''<0$, and recalling that $\gamma'(z)>0$, we know $\gamma''(z)>0$. Taking the derivative of $z^{\text{min}}(p)$ along with the above results, we can then determine that

$$\frac{\partial z^{\text{min}}(p)}{\partial p} = \gamma'(pU(\bar{\pi}))U'(\bar{\pi}) > 0$$

and $\frac{\partial^2 z^{\text{min}}(p)}{\partial p^2} = \gamma''(pU(\bar{\pi}))U'(\bar{\pi})^2 > 0$, confirming that indeed $z^{\text{min}}(p)$ is an increasing convex function of $p$. Proving that $z^{\text{min}}(0) = 0$ and $z^{\text{min}}(1) = 1$ is straightforward from equation (1).}

Therefore, the above analysis implies that $z^{\text{min}}(p) < p\bar{\pi}$ for all $p \in (0, 1)$, or that a risk-averse prosecutor is always willing to offer a plea bargain that is strictly less than the expected sentence from trial to all defendants, and moreover, that $p\bar{\pi} - z^{\text{min}}(p)$ is greatest for those $p$ relatively far from zero and one.

Intuitively, the stronger the case against a defendant, the smaller the discount the prosecutor is willing to offer the defendant for pleading guilty. However, for those cases where there is a great deal of uncertainty regarding the outcome of a trial (i.e. those defendants for whom $p$ is relatively far from zero and one), risk-averse prosecutors have a strong desire to mitigate this uncertainty, and therefore are willing to offer a plea bargain sentence substantially below the expected sentence from going to trial. On the other hand, in those cases where there is little uncertainty regarding the trial outcome (i.e. those defendants for whom $p$ is relatively close to zero or one), risk-averse prosecutors have little to gain from avoiding the small amount of uncertainly associated with a trial.
2.2 Defendant Behavior

Given defendant preferences are given by a function \( V(z) \), an expected utility maximizing defendant with strength of case \( p \) will never accept a plea bargain sentence \( z \) such that

\[
V(z) < pV(\pi),
\]

where the right-hand side of the above equation gives the defendant’s expected utility from going to trial. Similar to above with regard to prosecutors, equation (2) implicitly defines a function \( z^{\text{max}}(p) \), where an expected utility maximizing defendant will always choose to go to trial over accepting a plea bargain sentence any longer than \( z^{\text{max}}(p) \). Moreover, given our assumptions regarding the defendants’ utility function \( V \), it can easily be shown that \( z^{\text{max}}(0) = 0 \), \( z^{\text{max}}(1) = 1 \), and \( \frac{\partial z^{\text{max}}(p)}{\partial p} > 0 \), or that the maximum plea bargain sentence a defendant is willing to accept is increasing in the probability of his being convicted at trial.\(^6\)

The other important issue to consider, however, is the second derivative of \( z^{\text{max}}(p) \), or how the maximum plea sentence a defendant is willing to accept changes with the probability he will be convicted at trial. Not surprisingly, this second derivative depends on whether defendants are risk-averse or risk-loving. In particular, it can be shown that for risk-averse defendants, \( \frac{\partial^2 z^{\text{max}}(p)}{\partial p^2} < 0 \), or that the maximum acceptable plea sentence is an increasing concave function of his probability of conviction at trial.\(^7\)

\(^6\)From equation (2), we can define \( z^{\text{max}}(p) \) to equal

\[
z^{\text{max}}(p) = \phi(pV(\pi)),
\]

where \( \phi \) is the inverse of the defendant’s utility function \( V \). Given the defendant’s utility function \( V \) was assumed to be a decreasing function, its inverse \( \phi \) will also be a decreasing function. Therefore, further noting that \( V(\pi) < V(0) = 0 \), we can determine that \( \frac{\partial z^{\text{max}}(p)}{\partial p} = \phi'(pV(\pi))V(\pi) > 0 \).

\(^7\)Taking the second derivative of \( z^{\text{max}}(p) \) we get \( \frac{\partial^2 z^{\text{max}}(p)}{\partial p^2} = \phi''(pV(\pi))V(\pi)^2 \). Once again, note that if \( \phi \) is the inverse function of \( V \), then \( \phi'(z) = \frac{1}{V'(\phi(z))} \) and \( \phi''(z) = -\frac{1}{V'(\phi(z))^2} V''(\phi(z)) \phi'(z) \). Therefore, if \( V'' < 0 \), and recalling that \( \phi'(z) < 0 \), we know
$z^{\text{max}}(0) = 0$ and $z^{\text{max}}(1) = 1$, implies that the maximum plea sentence a risk-averse defendant is willing to accept is greater than his expected sentence from going to trial, or $z^{\text{max}}(0) > p\overline{z}$. Intuitively, similar to prosecutors, risk-averse defendants are willing to “pay” a little (in the sense of a longer expected sentence) to avoid the possibility of being convicted at trial and receiving the statutory sentence $\overline{z}$, with this willingness to pay being the greatest when uncertainty is the highest (i.e. $p$ lies far from zero or one).

On the other hand, as discussed above, it may actually be more plausible that $V'' > 0$, or that defendants are risk-loving in sentence length. In this case, it can be shown that $\frac{\partial^2 z^{\text{max}}(p)}{\partial p^2} > 0$. Therefore, when defendants are risk-loving, $z^{\text{max}}(p)$ is an increasing convex function of $p$. Given $z^{\text{max}}(1) = \overline{z}$ and $z^{\text{max}}(0) = 0$, this means $z^{\text{max}}(p) < p\overline{z}$ for all $p \in (0, 1)$, or that the maximum acceptable plea bargain for risk-loving defendants is strictly less than the expected sentence from trial. The intuition for this result is that risk-loving defendants prefer to gamble between receiving no sentence and a long sentence of $\overline{z}$, over a sure sentence of $p\overline{z}$, meaning the maximum sentence they would be willing to plead to must be somewhat lower than $p\overline{z}$. Moreover, this discount below $p\overline{z}$ must be greatest when uncertainty is highest, or when $p$ lies far from zero or one.

### 2.3 Resulting Sentences

Given $z^{\text{min}}(p)$ and $z^{\text{max}}(p)$ as derived above, it should be clear that a plea bargain can be reached if and only if $z^{\text{min}}(p) \leq z^{\text{max}}(p)$, or when the maximum plea bargain sentence a defendant is willing to accept is greater than or equal to the minimum plea bargain sentence a prosecutor is willing to offer. For ease of

\[ \phi''(z) < 0, \text{ meaning } \frac{\partial^2 z^{\text{max}}(p)}{\partial p^2} < 0.\]

\[8\text{As above, we know } \frac{\partial^2 z^{\text{max}}(p)}{\partial p^2} = \phi''(p V(\overline{z}))(V(\overline{z}))^2. \text{ Noting that if } \phi \text{ is the inverse function of } V, \text{ then } \phi'(z) = \frac{1}{V'(\phi(z))} \text{ and } \phi''(z) = -\frac{1}{V'(\phi(z))^2} V''(\phi(z))\phi'(z). \text{ Therefore, if } V'' > 0, \text{ and recalling that } \phi'(z) < 0, \text{ we know } \phi''(z) > 0, \text{ meaning } \frac{\partial^2 z^{\text{max}}(p)}{\partial p^2} > 0.\]
reference, let $z_{\alpha}^{\max}(p)$ denote the maximum plea bargain sentence a risk-averse defendant would accept, meaning $z_{\alpha}^{\max}(p)$ is an increasing concave function of $p$ that exceeds $p\bar{z}$ for all $p \in (0, 1)$. Similarly, let $z_{\ell}^{\max}(p)$ denote the maximum plea bargain sentence a risk-loving defendant would accept, meaning $z_{\ell}^{\max}(p)$ is an increasing convex function of $p$ that is less than $p\bar{z}$ for all $p \in (0, 1)$.

Given the above notation, note that for any given $p$, $z^{\min}(p) < p\bar{z} < z_{\alpha}^{\max}(p)$, meaning that if defendants are risk-averse, a plea bargain can always be struck. On the other hand, since both $z^{\min}(p) < p\bar{z}$ and $z_{\ell}^{\max}(p) < p\bar{z}$ for all $p \in (0, 1)$, a plea bargain agreement may or may not be attainable if defendants are risk-loving. In general, whether $z^{\min}(p) \leq z_{\ell}^{\max}(p)$ depends on the relative curvature of the prosecutor’s utility function $U$ versus that of the defendants utility function $V$. Hence, as long as prosecutors are risk-averse, then even if trials and incarceration were costless, there were no concerns about incarcerating innocent defendants, and no information regarding other crimes had to be coerced from defendants, it still may be optimal to resolve all cases via plea bargaining. However, if a defendant is risk-loving and his utility function $V$ exhibits relatively more curvature than the prosecutor’s utility function $U$, then a plea bargain cannot be struck even if a prosecutor is risk-averse. Intuitively, a plea bargain can be struck only if the marginal disutility a defendant incurs from additional time in jail is relatively high even if he has been there awhile.

Now let us consider how to model the sentence outcome in cases where a plea bargain can be struck. To keep things simple, let the relative bargaining power of the prosecutor be captured by a parameter $\rho \in [0, 1]$. Specifically, say that when the maximum plea sentence a defendant is willing to accept exceeds the minimum plea sentence the prosecutor is willing to offer, the resulting negotiated plea bargain sentence equals
\[ z^*(p) = \rho z_{\text{max}}(p) + (1 - \rho)z_{\text{min}}(p). \]

Hence, when \( \rho = 1 \), prosecutors have all the bargaining power, meaning they can make a take-it-or-leave-it offer, extracting all of the bargaining “rent” by ensuring that the defendant agrees to a plea bargain sentence of \( z_{\text{max}}(p) \). The opposite will be true when \( \rho = 0 \). When bargaining power is more equally balanced, \( \rho \) will lie strictly between zero and one, meaning the resulting plea bargain sentence will lie between \( z_{\text{min}}(p) \) and \( z_{\text{max}}(p) \).

Once again, let us first consider the case for when both prosecutors and defendants are risk-averse. As discussed above, when this is the case, \( z_{\text{min}}(p) < p\overline{z} < z_{\text{max}}(p) \), meaning the plea bargain sentence \( z^*(p) \) can be greater than, less than, or equal to the expected sentence from going to trial for a defendant with any given \( p \), depending on the bargaining power parameter \( \rho \), but essentially should be roughly centered around the expected sentence from trial \( p\overline{z} \).

Alternatively, when defendants are risk-loving, it will be the case that when plea bargains can be made, it must be true that \( z_{\text{min}}(p) \leq z_{\ell}^{\text{max}}(p) < p\overline{z} \), meaning \( z^*(p) < p\overline{z} \) regardless of the relative bargaining power \( \rho \). In words, if defendants foresee becoming acclimated to prison over time, or more generally, perceive that the marginal disutility of incarceration will be diminishing in magnitude as sentence length increases, any plea bargain sentences must necessarily fall short of the expected sentence from going to trial. In general, the less risk-averse are defendants, the more costly it becomes for prosecutors to “purchase insurance” against the risk of an acquittal. Indeed, once defendants can be classified as risk-loving, prosecutors will only be able to mitigate the risk of trial through plea bargaining if they offer a sentence less than the expected sentence from going to trial.

Finally, it is worth noting that since both \( z_{\text{min}}(p) \) and \( z_{\ell}^{\text{max}}(p) \) were shown
to be increasing convex functions of $p$, it follows that $z^*(p)$ must also be an increasing convex function of $p$ when defendants are risk-loving. Figure 1 reveals an important implication of this result. Specifically, the dashed line shows how the expected sentence from trial increases with $p$. The curve below shows how the plea bargain sentence $z^*(p)$ will change with $p$ when defendants are risk-loving. Note that the convex shape of the $z^*(p)$ function means that the greatest difference between the expected sentence at trial and the resulting plea bargain sentence will be for those defendants with moderate evidence against them (i.e. those defendants with a $p$ lying near $p_2$). Hence, the distribution of plea bargain sentences might be far more concentrated toward zero than the actual distribution of expected sentences from trial if a large fraction of defendants have “moderate” evidence against them.

Figures 2a and 2b show reveal this point graphically. Specifically, these figures show the results from a simple parameterized simulation of the model where I assume prosecutor’s preferences are of the form $U(z) = \frac{z^{1-\eta}}{1-\eta}$ and defendant’s preferences are of the form $V(z) = -\frac{z^{1-\sigma}}{1-\sigma}$. Note that under these formulations, as $\eta$ increases the marginal benefit to the prosecutor from increasing a sentence becomes smaller and smaller the longer the imposed sentence, or equivalently, the prosecutor becomes increasingly risk-averse. Similarly, as $\sigma$ increases, defendants become increasingly risk-loving. For the purposes of this simulation, let $\eta$ and $\sigma$ both equal 0.6, let $\rho = 0.5$ meaning neither party has dramatically greater bargaining power, and for sake of argument, say that the statutory sentence, $\tau$, is 20 years. Finally, assume $p$ has a bell shaped distribution centered around 0.5. In this simple parameterization, a plea bargain can be struck for defendants of any given $p$ (i.e. $z_{\text{min}}(p) < z_{\text{max}}(p)$ for all $p$). Comparing Figures 2a and 2b, we can see the large difference between the distribution of expected sentences if all defendants had to go to trial and the distribution of sentences
that would arise under optimal plea bargaining. These differences are due to the non-linear shape of the plea bargain sentence function $z^*(p)$ that ensures substantially discounted sentences for those defendants with moderate evidence against them.

In summary, this parameterization shows that if the probability of conviction at trial is substantially different than one or zero for a large fraction of defendants, then even if there are no frictions in the judicial system, it may still be optimal for society to resolve most cases through plea bargaining, with these plea bargaining sentences not only falling well short of the statutory sentence that would be imposed upon conviction at trial, but also well short of even the expected sentences from trial.

3 Discussion

The model developed above reveals that plea bargaining may have a substantial impact on judicial outcomes even in a world where trials and incarceration are costless, and prosecutors are not charged with the role of differentiating between guilty and innocent defendants or with obtaining further information from defendants about other crimes or criminals. In particular, risk-averse preferences among citizens and prosecutors regarding the incarceration of defendants mean society/prosecutors may find it optimal to ensure a sentence through offering a plea bargain that is acceptable to the defendant, even if this means offering a substantially discounted sentence from what the defendant would receive if convicted at trial.

While the above result is not particularly surprising, and indeed has been raised before by Grossman and Katz (1983), this analysis extends the discussion by allowing for more general defendant preferences. In particular, Grossman and Katz only examine the case where defendants are also risk-averse with respect
to sentence length. As shown in the analysis above, under this formulation, all cases should theoretically be resolved via plea bargaining, and the agreed upon plea sentence should lie in rough proximity to the expected sentence from trial.

However, the analysis above also examines the consequences of risk-loving preferences among defendants—an arguably more realistic assumption if one believes prisoners become acclimated to being incarcerated over time. Allowing for such defendant preferences has some important consequences. In particular, no longer will it necessarily be true that all cases should theoretically be resolved via plea bargains, as risk-loving defendants may require bigger sentence discounts in a plea bargain than optimizing prosecutors are willing to offer. However, when mutually acceptable plea bargains are possible with risk-loving defendants, it will necessarily be the case that the agreed upon plea sentence will lie short of the expected sentence from going to trial, and possibly far short for those defendants with only moderate cases against them.

While the analysis presented here is primarily meant to be a theoretical exercise that considers the potential role for plea bargaining, as well as what this may mean for sentencing distributions, in a world where many of the standard justifications for plea bargaining are not relevant, it is still interesting to consider this model in the context of actual sentencing outcomes as revealed by data from the State Court Processing Statistics (SCPS)\(^9\) In particular, a first thing to note is that while the fraction of cases resolved via plea bargaining is very high (roughly 92 percent), there are still 8 percent of cases in which an acceptable plea bargain did not arise, which runs counter to the one-hundred percent plea bargaining rate that would occur under Grossman and Katz’ (1983) assumption

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\(^9\)The SCPS constitute a representative sample of all individuals arrested for felonies in a given month in the largest 75 counties bi-annually between 1990-2002, and contains information on the arrest charge, criminal history, and other key aspects of each case, as well as information on adjudication outcomes and sentencing. Cases still pending or that had missing adjudication or sentencing information were not included in my sample. Also, KY counties were dropped due to too few cases going to trial. This left a sample of 54,789 cases. Data made available through ICPSR.
that all defendants are risk-averse. Furthermore, the SCPS data shows that the mean sentence length for those who resolve their case through plea bargaining is just over 17 months, compared to a mean sentence length of almost 45 months for those who go to trial (which includes those who are acquitted and therefore receive a sentence length of zero). Given the mean conviction rate of those who go to trial is 0.80, this suggests that plea sentences lie well short of the expected sentence from trial unless the probability of conviction at trial for those who plea bargain is less than half the probability of conviction of those who go to trial.\footnote{To attempt to account for differences in characteristics between those who go to trial and those who plea bargain, I also compared the weighted mean sentence for those who go to trial versus those who plea bargain, where each individual is weighted by his “propensity score” for going to trial, where propensity scores were obtained by estimating a probit specification using a binary variable indicating whether the defendant went to trial as the dependent variable and right-hand side variables included dummies for arrest charge, an indicator for two or more previous felony convictions, an indicator for one previous felony conviction, dummies for race and gender, state specific time trends, age and age squared, a dummy indicating a private attorney, dummy indicators for whether the defendant was on probation or parole at the time of arrest, dummies indicating whether the defendant was a fugitive or in custody at the time of arrest, and a dummy indicating the defendant was re-arrested before his/her original case was adjudicated. Results using these propensity score weights were qualitatively similar to the unweighted difference in means discussed above.} While these results likely reflect a variety of different prosecutorial and defendant motivations, they are at least consistent with prosecutors being risk-averse but a substantial fraction of defendants being risk-loving.
References


Figure 1 - Plea Bargain Sentence $z^*(p)$ versus Expected Sentence from Trial for three levels of $p$, given risk-loving defendants
Figure 2a - Distribution of Expected Sentences From Trial
(p distributed normally with mean = 0.5 and s.d. = 0.15)
Figure 2b - Distribution of Plea Bargain Sentences (given risk-loving defendants)

(p distributed normally with mean = 0.5 and s.d. = 0.15)