Prof. Asuman Aksoy
Math Analysis I
HW 2
Due 02/02/2012

1)

a) Let $f : [\frac{\pi}{2}, \frac{3\pi}{2}] \to [-1, 1]$ be given by $f(x) = sinx$. Prove or disprove: f is a bijection, and its inverse function is arcsinx.

b) Find $f(E)$ and $f^{-1}(E)$ for $f(x) = 2 - 3x$ and $E = (-1, 2)$

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2) Suppose $f : A \to B$ and $g : B \to C$ are functions show that

a) If both f and g are one-to-one, then $g \circ f$ is one-to-one.

b) If both f and g are onto, then $g \circ f$ is onto.

c) If both f and g are bijection, then $g \circ f$ is bijection.

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3)
For a function $f : X \longrightarrow Y$, show that the following statements are equivalent.

a) f is one-to-one

b) $f(A \cap B) = f(A) \cap f(B)$ holds for all $A, B \in \mathcal{P}(X)$

Hint: For $a) \Rightarrow b)$ you can assume $f(A \cap B) \subseteq f(A) \cap f(B)$.
For $b) \Rightarrow a)$ consider $A = \{a\}$ and $B = \{b\}$.

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4) For an arbitrary function $f : X \longrightarrow Y$, prove the following identities:

a) $f^{-1}\left(\bigcup_{i\in I} B_i\right) = \bigcup_{i\in I} f^{-1}(B_i)$

b) $f^{-1}\left(\bigcap_{i\in I} B_i\right) = \bigcap_{i\in I} f^{-1}(B_i)$

c) $f^{-1}(B^c) = [f^{-1}(B)]^c$

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5)

a) Show that if r is rational ($r \neq 0$) and x is irrational, then $r + x$ and rx are irrational.

b) Show that there is no rational number whose square is 12

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6) Let I is an interval and $f : I \to \mathbb{R}$ is a differentiable function. Prove that if the derivative of f either always positive on I, or always negative on I, then f is one-to-one on I.

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7)

a) Prove that two real numbers a and b are equal if and only if for every real number $\epsilon > 0$ it follows that $|a - b| < \epsilon$.

b) Use the triangle inequality to establish the inequalities:

- $||a| - |b|| \leq |a - b|$
- $|a - b| \leq |a| + |b|$.

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