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★**Nonstandard methods in fixed point theory.**

With an introduction by W. A. Kirk.

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The question whether every weakly compact convex subset of a Banach space has the fixed point property for nonexpansive mappings remained open for a long time [the reviewer, Amer. Math. Monthly **83** (1976), no. 4, 266–268; ibid. **87** (1980), no. 4, 292–294]. It has been answered in the negative by D. E. Alspach [Proc. Amer. Math. Soc. **82** (1981), no. 3, 423–424; MR0612733 (82j:47070)] for the Lebesgue space $L^1[0,1]$. On the positive side, it has been shown by B. Maurey [in *Séminaire d'Analyse Fonctionnelle, 1980–1981*, Exp. No. VIII, École Polytech., Palaiseau, 1981; MR0659309 (83h:47041)] that all weakly compact convex subsets of reflexive subspaces of $L^1[0,1]$ have the fixed point property for nonexpansive mappings. Using ultraproducts of Banach spaces [D. Dacunha-Castelle and J.-L. Krivine, Studia Math. **41** (1972), 315–334; MR0305035 (46 #4165)], Maurey also proved that all the weakly compact convex subsets of the sequence space c_0, the Hardy space H^1, and the spaces X_r, $1<r<\infty$, possess this fixed point property. (X_r is the space l^2 renormed by $|x|_r=\max(||x||_2, r||x||_\infty)$, where $||\cdot||_2$ denotes the l^2 norm and $||\cdot||_\infty$ denotes the l^∞ norm.)

The main purpose of the book under review is to present these results of Maurey. To this end, the text begins with an exposition of Schauder bases, filters, and ultrafilters. The second chapter is devoted to the set-theoretic and Banach space ultraproduct constructions. Finally, Maurey's results, as well as several related fixed-point theorems (e.g. those of P. K. Lin [Pacific J. Math. **116** (1985), no. 1, 69–76; MR0769823 (86c:47075)] concerning spaces with unconditional Schauder bases), are established in Chapter 3. For other aspects of fixed-point theory for nonexpansive mappings in Banach and other spaces see a book by K. Goebel and the reviewer [*Uniform convexity, hyperbolic geometry, and nonexpansive mappings*, Dekker, New York, 1984; MR0744194 (86d:58012)], the new book by Goebel and W. A. Kirk [*Topics in metric fixed point theory*, Cambridge Univ. Press, Cambridge, 1990], and the recent papers by Khamsi, W. M. Kozlowski and the reviewer [Nonlinear Anal. **14** (1990), no. 11, 935–953; MR1058415 (91d:47042)], and by the reviewer and I. Shafrir [same journal **15** (1990), no. 6, 537–558]. It is still not known whether every weakly compact (equivalently, bounded closed) convex subset of a reflexive Banach space has the fixed-point property for nonexpansive mappings.

Reviewed by *Simeon Reich*