MR870973 (88h:47028) 47B10 (46A50)
Aksoy, Asuman (1-OAKL); **Nakamura, Masatoshi [Nakamura, Masatoshi[1]]** (J-KOBEE)
The approximation numbers $\gamma_n(T)$ and Q-precompactness.
Math. Japon. **31** (1986), *no. 6,* 827–840.

Let E be a (real or complex) topological linear space; for each $n \in \mathbf{N}$ let $\mathcal{A}_n = \mathcal{A}_n(E)$ be a family of subsets of E satisfying the following conditions: (1) $\{0\} = \mathcal{A}_0 \subset \mathcal{A}_1 \subset \cdots \subset \mathcal{A}_n \subset \cdots$, (2) $\lambda\mathcal{A}_n \subset \mathcal{A}_n$ for every $\lambda \in K = \mathbf{R}$ or $\mathbf{C}$ and $n \in \mathbf{N}$, (3) $\mathcal{A}_n + \mathcal{A}_m \subset \mathcal{A}_{n+m}$ for every $n, m \in \mathbf{N}$. Then $(E, (\mathcal{A}_n(E))_{n\in\mathbf{N}})$ is called an approximation scheme on E. If for each $n \in \mathbf{N}$ the family $\mathcal{A}_n(E)$ consists of one subset A_n of E, the concept of an approximation scheme introduced by A. Pietsch [J. Approx. Theory **32** (1981), no. 2, 115–134; MR0633697 (83b:41043)] will be obtained. The authors generalize the concepts of approximation numbers $\alpha_n(T)$ [resp. Kolmogorov numbers $\delta_n(T)$] for a given continuous linear operator T between complete p-normed spaces, $0 < p \leq 1$. They discuss the relation between $\alpha_n(T)$ and $\delta_n(T)$ and obtain results similar to those on Kolmogorov numbers. For example: let E be a complete p-normed space, F a complete q-normed space $(0 < p, q \leq 1)$ and $T \in L(E, F)$, then $\alpha_n(TQ_E) = \delta_n(T)$, where Q_E is the metric surjection of l^p_I onto E defined by $Q_E(\xi_x) := \sum \xi_x x$ for $(\xi_x)_{x\in I} \in l^p_I$, $I = U_{E'}$. They also consider precompactness of a bounded subset with respect to the approximation scheme $\mathcal{A}_n(E)$ and prove a Dieudonné-Schwartz type characterization of $\mathcal{A}_n(E)$-precompact sets in a p-normed space.

Reviewed by *Bernd Rosenberger*