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MR2428603 (2009h:54046) 54E45 (05C05 47H09 54E35)
Aksoy, Asuman G. (1-CMKC); **Borman, Matthew S.** (1-CHI);
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Compactness and measures of noncompactness in metric trees. (English summary)

Banach and function spaces II, 277–292, *Yokohama Publ.*, *Yokohama*, 2008.

A metric segment from x to y, denoted by $[x,y]$, is a subset of a metric space (X,d) such that $h_{x,y}\colon z\to d(x,z)$ is an isometry from $[x,y]$ onto $[0,d(x,y)]\subset \mathbf{R}$, and an open metric segment from x to y, denoted by (x,y), is $[x,y]\setminus\{x,y\}$, where $\mathbf{R}$ is the set of real numbers with the Euclidean metric. (M,d), a metric space, is called a metric tree (T-theory or $\mathbf{R}$-tree) if there exists a unique metric segment from x to y, and the equality $[x,z]\cap[z,y]=\{z\}$ implies the equality $[x,z]\cup[z,y]=[x,y]$ for all $x,y,z\in M$. The study of metric trees began with J. Tits [in *Contributions to algebra (collection of papers dedicated to Ellis Kolchin)*, 377–388, Academic Press, New York, 1977; MR0578488 (58 #28205)] and since then, applications have been found for metric trees within many fields of mathematics such as geometry, topology, and group theory [M. Bestvina, in *Handbook of geometric topology*, 55–91, North-Holland, Amsterdam, 2002; MR1886668 (2003b:20040)], computer science [I. Bartolini, P. Ciaccia and M. Patella, in *String processing and information retrieval*, 423–431, Lecture Notes in Comput. Sci., 2476, Springer, Berlin 2002], and biology and medicine [C. Semple and M. A. Steel, *Phylogenetics*, Oxford Univ. Press, Oxford, 2003; MR2060009 (2005g:92024)].

The authors first give some basic properties of metric segments in metric trees using the known results for metric segments in metric spaces. They prove that $M=\bigcup_{f\in F}[a,f]$ for every compact metric tree M and any point a of M where F is the set of final points of M given by $F:=\{f\in M|f\notin(x,y)$ for all $x,y\in M\}$. Necessary and sufficient conditions for a metric tree to be compact are given as $M=\bigcup_{f\in F}[a,f]$ for all $a\in M$ and the compactness of the closure of F. They show that $\alpha(A)=2\beta(A)$ for every bounded subset A of M where $\alpha(A):=\inf\{b>0|A\subset\bigcup_{j=1}^n E_j$ for some $E_j\subset A,\ \operatorname{diam}|(E_j)\le b\}$ and $\beta(A):=\inf\{b>0|A\subset\bigcup_{j=1}^n B(x_j,b)$ for some $x_j\in M\}$.

A continuous map T between metric trees M and N is called k-set-contractive if $\alpha(T(A))\le k\alpha(A)$, and is called k-ball-contractive if $\beta(T(A))\le k\beta(A)$ for every bounded subset A of M where k is a non-negative real number. They prove that a function from a subset of a metric tree to a metric tree is k-set-contractive if and only if it is k-ball-contractive and that the Lifschitz characteristic of M, denoted by $\kappa(M)$, defined by $\sup\{b>0|\ b$ is Lifschitz for $M\}$ is equal to 2 for any metric tree where a positive real number b is called Lifschitz for M if there exists $a>1$ such that for all $x,y\in M$, $r>0$, the inequality $d(x,y)>r$ implies that there exists $z\in M$ such that $B_c(x;ar)\cap B_c(y;br)\subset B_c(z;r)$, where $B_c(z,r)$ denotes the closed ball centered at z with radius r.

{For the entire collection see MR2446214 (2009g:46001)}

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