

MR2283573 (2008f:54050) 54H25 (47H09 47H10 54E40 55M20)

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Fixed points of uniformly Lipschitzian mappings in metric trees. (English summary)

Sci. Math. Jpn. **65** (2007), *no. 1*, 31–41.

A metric space (M, d) is called a metric tree if for every pair of distinct points $x, y \in M$ there is a unique arc between x and y and this arc is isometric to an interval in \mathbb{R} . A mapping $T: M \rightarrow M$ is said to be eventually uniformly Lipschitzian if $\sigma(T) = \limsup_{n \rightarrow \infty} \text{Lip}(T^n) < \infty$, where $\text{Lip}(T^n)$ is the Lipschitz constant for the n -th iteration of T .

The main theorem (Theorem 3.3) of the paper under review may be stated as follows: If M is a non-empty complete metric tree, and $T: M \rightarrow M$ is an eventually uniformly Lipschitzian mapping with bounded orbits and $\sigma(T) < 2$, then T has a fixed point.

As a corollary the authors obtain the following stability result for the fixed point property (Theorem 3.4): Let (M, d) be a non-empty complete metric tree, and d^* an equivalent distance such that $d \leq d^* \leq bd$, where $b < 2$. If a mapping $T: M \rightarrow M$ is non-expansive with respect to d^* and has bounded orbits, then T has a fixed point.

Reviewed by *Jerzy Krzempek*

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