Interpolation of the essential spectrum and the essential norm. (English summary)

Let \((X_0, X_1)\) be a compatible pair of Banach spaces and let \(T\) be an operator that acts boundedly on both \(X_0\) and \(X_1\). Let \(T_{\theta}\) \((0 \leq \theta \leq 1)\) be the corresponding operator on the interpolation space \((X_0, X_1)_{\theta}\). By [E. Albrecht and V. Müller, Proc. Amer. Math. Soc. 129 (2001), no. 3, 807–814 (electronic); MR1804050 (2001j:47001)], the set-valued function \(\theta \mapsto \sigma(T_{\theta})\) is in general discontinuous on \((0, 1)\) (although the spectral radius is continuous).

The present paper observes that in the above-mentioned example of Albrecht and reviewer the essential spectrum is also discontinuous. Further, the logarithmic convexity (up to a multiplicative constant) of the essential norm under real interpolation is studied and proved under certain compact approximation conditions. However, it is conjectured that the logarithmic convexity of the essential norm fails in general (although there are other measures of non-compactness that are logarithmically convex).

{For the entire collection see MR2173290 (2006e:46001)}

Reviewed by Vladimir Müller

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