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Q-compact sets and Q-compact maps.

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Let E be a Banach space and let $Q=(Q_n(E))_{n\in\mathbf{N}}$ be an approximation scheme [see, e.g., P. L. Butzer and K. Scherer, *Approximationsprozesse und Interpolationsmethoden*, Bibliographisches Inst., Mannheim, 1968; MR0259461 (41 #4099)] on E. The author studies compactness relative to Q for bounded sets and linear operators. The generalized Kolmogorov numbers $\delta_n(D;Q)=\inf\{r>0\colon D\subset A+rB_E,\ A\in Q_n(E)\}$ are introduced for bounded sets $D\subset E$, where B_E is the closed unit ball of E. The set D is said to be Q-compact if $\lim_{n\to\infty}\delta_n(D;Q)=0$. The Q-compact sets are characterized as subsets of the closed convex hull of certain uniform null-sequences. It is shown that $\delta_n(D;Q)$ converges as $n\to\infty$ to a corresponding measure of non-Q-compactness for all sets D. This setting reduces to that of the usual Kolmogorov numbers and the ball measure of noncompactness if $Q_n(E)$ consists of the subspaces of E with dimension less than n.

Reviewed by *Hans-Olav Tylli*