Math 151—Probability.

FINAL EXAM
O'NEILL
DEC. 14, 2005

Name: ____

Directions: No books or notes are allowed. Do not share calculators. Write only on the paper provided.

Question	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

1. Two candidates, A and B are running for President. To predict the outcome of the election, n people are selected at random and independently of the others and asked their choice. The predicted percentage p_A of votes for candidate A is given by dividing the number of people in the sample who say they will vote for A, by n.

   How many people need to be in the sample so that the probability that the predicted percentage p_A is correct to within .02 is at least .95?

   Hints: Let $X_i = 1$ if the i^{th} sample member will vote for A and 0 otherwise. You will need the facts that $p(1-p) \leq \frac{1}{4}$ for $0 \leq p \leq 1$ and that $\Phi(1.96) \approx .975$.

2. The moment generating function of a standard normal random variable is $M(t) = e^{t^2/2}$

   (a) What is the moment generating function of a random variable with an $N(\mu, \sigma^2)$ distribution?

   (b) If $X_1, X_2, \ldots, X_n$ are independent normal random variables with means μ_i and variances σ_i^2, show that

$$Y = \sum_{i=1}^{n} \alpha_i X_i,$$

   where the α_i are scalars, is normally distributed, and find its mean and variance.

3. A rod of unit length is broken at random into two pieces. Let R be the ratio of the shorter piece to the longer piece. Find the density function of R and also find the mean and variance of R.

4. (a) Let N_1 and N_2 be independent Poisson random variables with parameters λ_1 and λ_2. Show that the distribution of $N_1 + N_2$ is Poisson with parameter $\lambda_1 + \lambda_2$

(b) If blue stars occur randomly in a region of space at a rate of .002 per cubic parsec and white stars occur in the same region at the rate of .005 per cubic parsec, and no other types occur, what is the probability that a randomly selected region of size 1000 cubic parsecs contains 5 stars?

4. Find the probability density for the distance from an event to its nearest neighbor for a Poisson Process in the plane.

4. In 3 dimensional space.

5. A box has three coins. One has two heads, one has two tails, and the other is a fair coin with one head and one tail. A coin is chosen at random, is plipped, and comes up heads.

   (a) What is the probability that the coin chosen is the two headed coin?

   (b) What is the probability that if it is thrown another time it will come up heads?

6. You draw five cards from a standard 52 card deck.

   (a) What is the probability that you have three of a kind?

   (b) What is the probability that you have no pair? (you may count straights and flushes as "no pair")

7. Suppose that X and Y have the joint density function

$$f(x,y) = c\sqrt{1-x^2-y^2}, \qquad x^2+y^2 \leq 1$$

(a) Find c.

(b) Find $P(X^2+Y^2 \leq \frac{1}{2})$

(c) Find the marginal densities of X and Y. (It may be helpful to sketch the joint density and to think geometrically)

(d) Find the conditional densities.

8. Let X and Y have the joint density

$$f(x, y) = e^{-y}, \qquad 0 \le x \le y.$$

(a) Find $\text{Cov}(X, Y)$ and the correlation of X and Y.

(b) Find $\text{E}(X|Y = y)$ and $\text{E}(Y|X = x)$.

(c) Find the density functions of the random variables $\text{E}(X|Y)$ and $\text{E}(Y|X)$.

9. Find the joint densities of $X+Y$ and $\frac{X}{Y}$, where X and Y are independent exponential random variables with parameter λ. Show that $X+Y$ and $\frac{X}{Y}$ are independent.