Math 151—Probability.

4:15 PM Section

Monday, October 13, 2008

Name: ______________________________

Directions: No books or notes are allowed. You may use a calculator but can get full credit without one.

Question	Points
1	
2	
3	
4	
5	
6	
7	
Total	

1. A candy factory has an endless supply of red,orange, yellow, green, blue and violet jelly beans. The factory packages the jelly beans into jars of 100 jelly beans each.
   (a) As a marketing gimmick, the factory guarantees that no two jars have the same color distribution. What is the maximum number of jars that the factory can produce?
   (b) If each jar is also (in addition to the claim made above) required to have at least one jelly bean of each color, what is the maximum number of jars that can be produced?

2. The joint probability mass function of a bivariate random variable (X, Y) is given by

$$p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j) & x_i = 1, 2; y_j = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of k.
   (b) Find the marginal pmf's of X and Y.
   (c) Are X and Y independent?

3. A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set
$$\{A, A-, B+, B, B-, C+, C, C-\}$$
with equal probability, independent of the other papers. How many papers do you expect to hand in before you receive all the possible grades at least once?

4. A company producing electric relays has three manufacturing plants producing 50, 30 and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are, respectively, 0.02, 0.05, and 0.01.
   (a) If a relay is selected at random from the output of the company, what is the probability that it is defective?
   (b) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?

5. Let X be a Poisson random variable with parameter $\lambda > 0$. Find $E(e^{tX})$ as a function of the real variable t.

   Recall that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

6. A random variable X is called a *Laplace* random variable if its probability distribution function is given by

$$f_X(x) = ke^{-\lambda|x|} \qquad \lambda > 0, \qquad -\infty < x < \infty$$

where k is a constant.

(a) Find the value of k.

(b) Find the cumulative distribution function of X.

(c) Find the mean and variance of X.

7. Suppose that conditional on N, X has a binomial distribution with N trials and probability p of success, and that N is a binomial random variable with m trials and probability r of success.
   (a) Find an explicit expression in terms of n, m, p, r for the unconditional distribution of X. (I have used n for the value of N.)
   (b) Use your expression from part (a), (or some other method), to show that X is binomial(m, pr).