

Math 151—Probability.
1:15 PM SECTION
WEDNESDAY, OCTOBER 10, 2007

Name: _____

Directions: No books or notes are allowed. You may use a calculator but can get full credit without one.

Question	Points
1	
2	
3	
4	
5	
6	
7	
8	
Total	

1. I have three painted dice. The first has one red face and five green faces. The second has three red faces and three green faces. The third has 5 red faces and one green face. I pick one dice at random and roll it twice. Let R_1 denote the event that the first roll shows a red face. Let R_2 be the event that the second roll shows a red face.
 - (a) Find $P(R_1)$.
 - (b) Are R_1 and R_2 independent? You must prove your answer.

2. In a certain state lottery, the probability of winning is $\frac{1}{9}$. A certain player purchases one ticket per week starting on her 21st birthday.
- (a) What is the probability that the player's first win doesn't occur until after the fourth week.
 - (b) What is the probability that her third win occurs in the 27th week.
 - (c) What is the probability that her third win occurred in the 27th week given that she lost in the first four weeks.

3. A standard deck consists of 52 cards. Four of the cards are aces. Cards are dealt from the deck at random without replacement until two aces have appeared.

For $i = 1, 2, 3, 4$, let X_i be the total number of cards dealt until the i^{th} ace appears.

- (a) Find $P(X_1 = 1, X_2 = 5)$.
- (b) Find $P(X_1 = 1 | X_2 = 5)$.
- (c) Find $P(X_4 = 52)$.

4. In a certain card room, 40,000 hands of poker are dealt each night between 5pm and 12 am. In each hand, there is a probability of 7×10^{-8} of a royal straight flush being dealt to some player in the hand. The casino gives a large cash prize to any player dealt a royal straight flush during these hours and is therefore interested in knowing (in some practically useful way):
- (a) What is the chance of there being no royal straight flush dealt on a given night.
 - (b) What is the chance of more than 5 royal straight flushes being dealt in a a given 30 day period.

5. Suppose that a virus has an incidence in the general susceptible population of $\frac{1}{250}$. Suppose also that a first stage screening for the virus has a false positive rate of 4%. If the rate of false negative tests is negligible, what is the probability that a patient who tests positive has the virus?

6. Let $F(x) = 1 - e^{-\alpha x^\beta}$ for $x \geq 0$, $\alpha > 0$, $\beta > 0$, and let $F(x) = 0$ for $x < 0$.
- (a) Show that F is a cdf, and find the corresponding density.
 - (b) Find the median of a random variable with this density.

7. A thoroughly honest game-show host has placed a car behind one of three doors. There is a goat behind each of the other doors. You have no prior knowledge that allows you to distinguish among the doors. "First you point toward a door," he says. "Then I'll open one of the other doors to reveal a goat. After I've shown you the goat, you make your final choice whether to stick with your initial choice of doors, or to switch to the remaining door. You win whatever is behind the door." You begin by pointing to door number 1. The host shows you that door number 3 has a goat.

Do the player's chances of getting the car increase by switching to Door 2? You must prove your answer.

8. Two dice are rolled a 100 times and the number of double sixes is counted.
- (a) What is the exact probability that there will be 7 occurrences of a double six?
 - (b) What is the Poisson approximation for the same probability?