Stochastic Computation Techniques

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What is Stochastic Computation?

Deterministic Algorithms fixed run time, always give correct output.

StoCom Algorithms involve randomness (named after famous gambling locations):

- **Monte Carlo**: Output is random.
- **Las Vegas**: Running time is random.
- **Atlantic City**: Fixed probability, output correct.
Examples

Las Vegas

Quicksort
Uses random choices
Run time random
At end, sorted data

Atlantic City

Primality Testing
Small chance of failure
Can make failure chance arbitrarily small
Monte Carlo

Monte Carlo most common:

- Roughly 1/3 of all computer time devoted to Monte Carlo algorithms

- Simple Example: Shuffling Cards

Goal of shuffling:

- Generate permutations of $n$ cards uniformly at random (standard deck $n=52$)

- Riffle shuffling fast

- Pass shuffling slow

- 52 card pickup
Monte Carlo Markov chain (MCMC)

Markov chain approach

- Make random changes get sequence: 
  \[ X_1, X_2, X_3, \ldots \]
- Memoryless (6\textsuperscript{th} shuffled state only depends on 5\textsuperscript{th} state, not 1\textsuperscript{st} through 4\textsuperscript{th})
  \[
  P(X_{t+1} | X_1, \ldots, X_t) = P(X_{t+1} | X_t)
  \]
- Eventually state close to “stationary” 
  \[ X_t \] almost uniform over permutations
The Million Dollar Question

How long until $X_t$ close to stationary? (How many times must I shuffle?)

- Riffle Shuffle: $O(\ln n)$
- Pass Shuffle: $O(n^2)$
- 52 Card Pickup: once (computers use)
### One type of problem

Contingency Tables

<table>
<thead>
<tr>
<th></th>
<th>Colgate</th>
<th>Aqua-Fresh</th>
<th>Crest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oregon</td>
<td>5</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>35</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>40</td>
<td>119</td>
</tr>
<tr>
<td>Washington</td>
<td>40</td>
<td></td>
<td>148</td>
</tr>
</tbody>
</table>
Does region affect toothpaste?

Pick random pair of rows and random pair of columns

Rows: 1, 2
Columns: 1, 3

Add and subtract 1 to four corners of rectangle

Keeps row and column totals the same
Questions...

Can we get to any table this way?
Yes

What is the mixing time?
Unknown

Is another set of moves faster?
Unknown