Perfect Simulation of Matérn Type III Repulsive Point Processes

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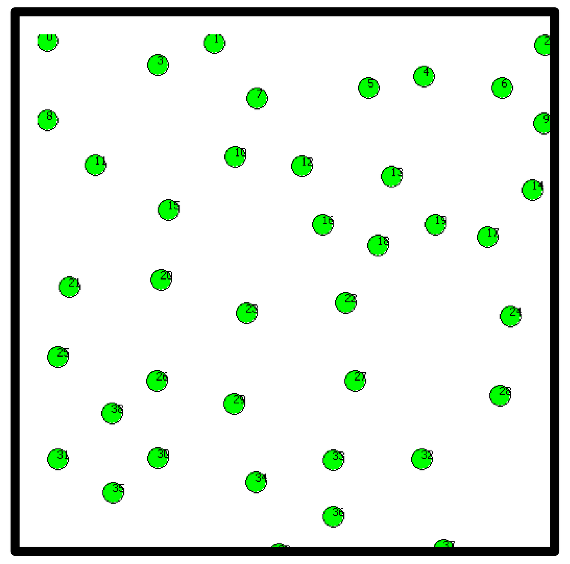
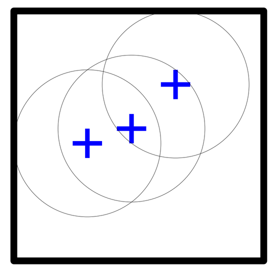
Overview



Spatial data are often more dispersed than would be expected if the points were independently placed. Such data can be modeled with *repulsive point processes*, where the points appear as if they are repelling one another. Various models have been created to deal with this phenomenon. Matérn created three algorithms that generate repulsive processes.

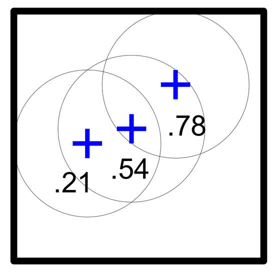
Here, Matérn Type III processes are used to approximate the likelihood and posterior values for data. Perfect simulation methods are used to draw auxilliary variables for each spatial point that are part of the type III process.

How to draw a Matérn Type III process



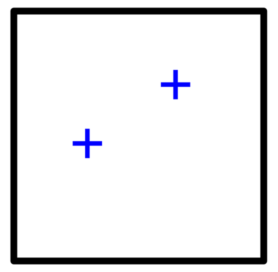
Step 1

Start with Poisson point process. (Put discs radius R around pts.)



Step 2

Assign each point a time stamp ~ Unif([0,1])



Step 3

Starting with smallest time stamp, remove all points within distance R and with a higher time stamp

Turning it into a density

Each point can be viewed as casting a shadow in space-time, where points in the shadow of other points get removed.

Leads to a density *g(x)* with respect to a unit rate Poisson point process.

$$A_\theta(x,t)=\lambda\cdot\text{volume of shadow in spacetime}$$

$$g_{\lambda,R}(x)\propto\int_{t\in[0,1]^n}\exp(A_\theta(x,t))\;dt$$

Goal: use *g(x)* to infer λ and *R*.

Metropolis-Hastings chain

With density, can formally write MH chain.

Calculating $A_\theta(x,t)$ difficult (even in 2D).

Use thinned Poisson point process to avoid computing MH ratio.

Choose a point *v*
Propose new time stamp for *v*
If increase shadow, accept
Else draw Pois. proc. in change in shadow
if no points, accept

Bounding chains

Standard problem with Markov chains—difficult to analyze mixing time.

Perfect simulation methods draw from stationary distribution without need to know mixing time.

Bounding chains + coupling from the past (CFTP) gives method for converting MH chain into perfect simulation algorithm.

Product Estimator

Disadvantage: slow

Advantage: immune to multimodal dist's

Take sequence of parameters:

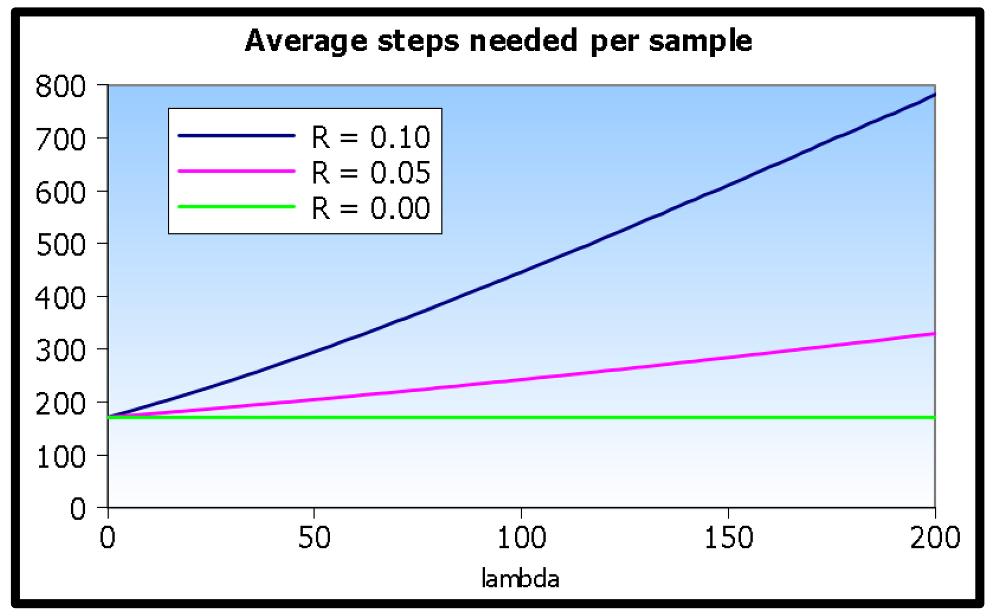
$$0<\lambda_1<\cdots<\lambda_k$$

Estimate ratios in sequence:

$$\frac{g_{\lambda_1,R}}{g_{0,R}}\frac{g_{\lambda_2,R}}{g_{\lambda_1,R}}\cdots\frac{g_{\lambda_k,R}}{g_{\lambda_{k-1},R}}=g_{\lambda_k,R}$$

Results for data set of 40 town locations in Spain

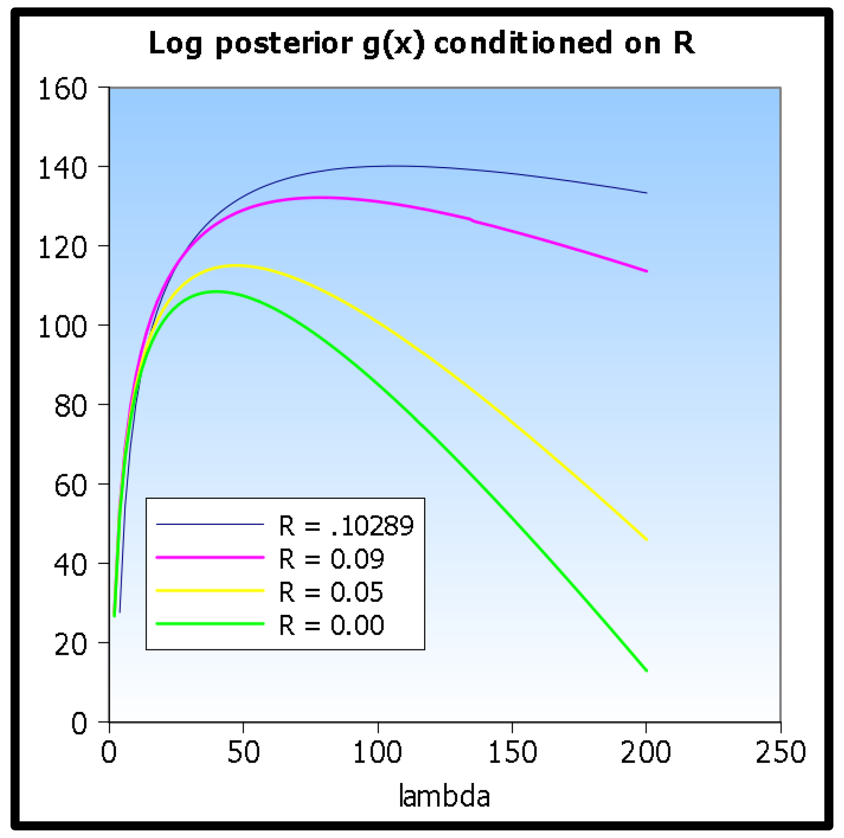
Algorithm running time seems to scale linearly in λ:

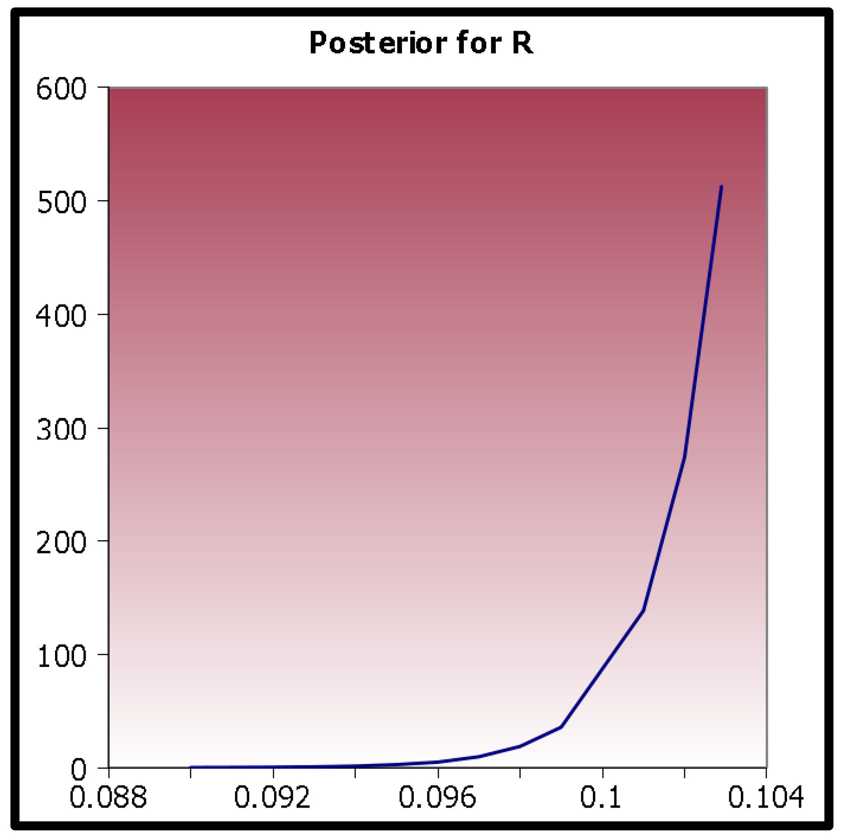


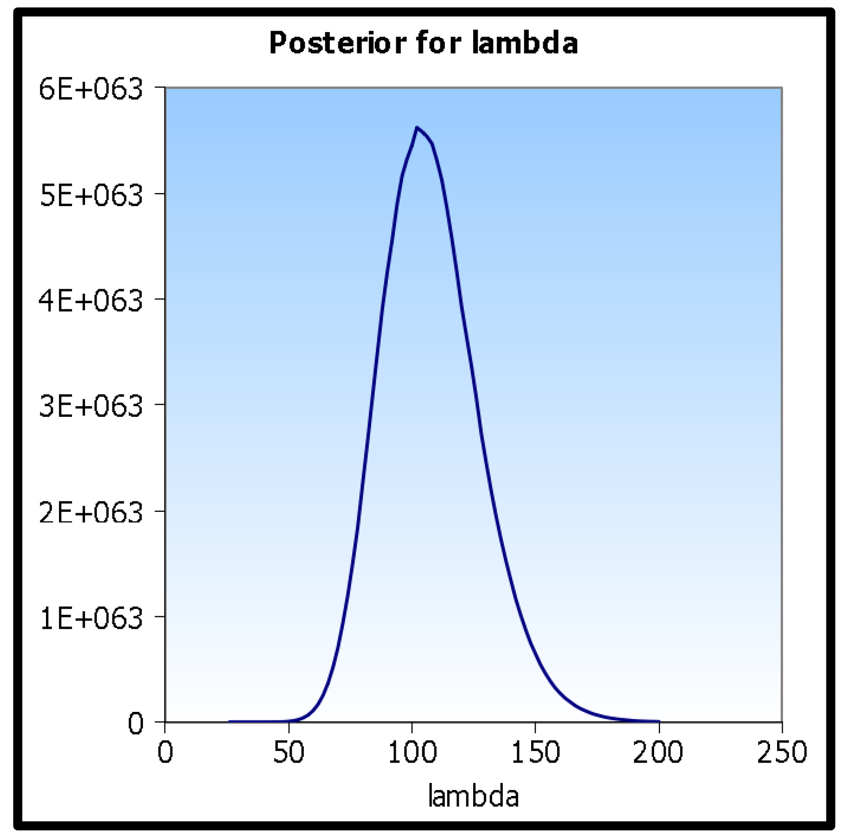
At each step in the M-H chain, a Poisson point process with intensity λ over the region close to a random point is generated in order to determine whether the proposed time stamp is accepted or rejected. Therefore each step takes time linear in λ as well. Combining, the total time to generate a set of time stamps will be quadratic in λ.

In contrast, the basic acceptance/rejection method for drawing perfect samples requires time exponential in λ.

Posterior of λ and *R* (Uses flat priors; note maximum over *i,j* of dist(x_i,x_j) is approx 0.10289)







Future Work

- Extend both model and algorithm to “soft core” where points penalized but not forbidden to fall within distance *R* of each other.
- Comparison to other repulsive models (area-interaction, Strauss process).
- Theoretical proof number of steps is linear in λ.

References

- Matérn (1986) *Spatial Variation*, vol. 36 of *Lecture Notes in Statistics.* New York, NY: Springer-Verlag 2nd ed.
- Propp & Wilson (1996) Exact sampling with coupled Markov chains and applications to statistical mechanics. *Random Structures & Algorithms*, 9(1-2):223-252
- Huber (2004) Perfect sampling using bounding chains. *Ann. of Applied Prob.*, 14(2):734-753