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# A Dissociation between Symbolic Number Knowledge and Analogue Magnitude Information

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Semantic understanding of numbers and related concepts can be dissociated from rote knowledge of arithmetic facts. However, distinctions among different kinds of semantic representations related to numbers have not been fully explored. Working with numbers and arithmetic requires representing semantic information that is both analogue (e.g., the approximate magnitude of a number) and symbolic (e.g., what ÷ means). In this article, the authors describe a patient (MC) who exhibits a dissociation between tasks that require symbolic number knowledge (e.g., knowledge of arithmetic symbols including numbers, knowledge of concepts related to numbers such as rounding) and tasks that require an analogue magnitude representation (e.g., comparing size or frequency). MC is impaired on a variety of tasks that require symbolic number knowledge, but her ability to represent and process analogue magnitude information is intact. Her deficit in symbolic number knowledge extends to a variety of concepts related to numbers (e.g., decimal points, Roman numerals, what a quartet is) but not to any other semantic categories that we have tested. These findings suggest that symbolic number knowl-

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edge is a functionally independent component of the number processing system, that it is category specific, and that it is anatomically and functionally distinct from magnitude representations. 

*Key Words:* acalculia; number processing; conceptual number knowledge; semantic memory; neuropsychology; dissociation; brain damage.

Researchers studying the functional organization of number processing sometimes distinguish between number knowledge that is procedural or automatic (e.g., arithmetic skills, retrieving multiplication facts), on the one hand, and number knowledge that is more semantic or conceptual (e.g., an understanding of the magnitude of a number, the knowledge that multiplication is commutative), on the other. However, the relationship between different kinds of semantic representations in the number domain has not been systematically investigated. In this article, we describe a neurological patient (MC) who is dramatically impaired on tasks that involve symbolic number knowledge but who is not impaired on tasks requiring the processing of analogue magnitude information. This dissociation suggests that different types of semantic representations used by the number system depend on different neural systems.

## *Dissociations between Mathematical Skills and Mathematical Understanding*

The distinction between mathematical skills, on the one hand, and mathematical understanding, on the other, is now well established (Delazer & Benke, 1997; Hittmair-Delazer, Sailer, & Benke, 1995; Hittmair-Delazer, Semenza, & Denes, 1994; McCloskey, Aliminosa, & Sokol, 1991; Sokol, McCloskey, & Cohen, 1989; Warrington, 1982). Retrieving memorized arithmetic facts (e.g., multiplication tables), applying certain general arithmetic rules (e.g., anything times zero equals zero), and implementing basic calculation procedures (e.g., the sequence of elementary operations involved in multidigit arithmetic) all might be considered mathematical skills (McCloskey et al., 1991). Such skills are assumed to be highly practiced and can be applied without a real appreciation of their underlying conceptual foundations. Real mathematical understanding, by contrast, is assumed to depend on knowledge that is more conceptual (e.g., the knowledge that multiplication is the same as repeated addition, the knowledge that addition and multiplication are commutative whereas subtraction and division are not). Conceptual number knowledge has been hypothesized to be distinct from mathematical skills, both functionally and neuroanatomically (Hittmair-Delazer et al., 1995).

Consistent with this distinction, a dissociation between knowledge of arithmetic tables and conceptual number knowledge has been found following brain damage. Warrington (1982) described a patient (DRC) who exhibited an impairment in retrieving arithmetic facts despite a preserved ability to describe the conceptual basis of each of the basic arithmetic operations. Hittmair-Delazer et al. (1994) described a patient (BE) who was impaired in recalling and using multiplication and division facts but who was able to solve multiplication and division problems by exploiting extensive conceptual knowledge that was intact. A similar patient (DA) exhibited intact processing of algebraic expressions and a good understanding of complex arithmetic text problems despite an inability to resolve simple arithmetic problems (Hittmair-Delazer et al., 1995). (For descriptions of other patients exhibiting similar dissociations, see McCloskey et al., 1991; McCloskey, Caramazza, & Basili, 1985; Sokol et al., 1989.)

Delazer and Benke (1997) recently described a patient (JG) who exhibited the converse dissociation, that is, a relative sparing of memorized arithmetic facts despite having lost conceptual knowledge underlying those same facts. Specifically, this patient made relatively few errors in single-digit multiplication tasks, whether presented verbally (59/64 correct) or visually (13/14 correct), but displayed virtually no understanding of multiplication at a conceptual level. For example, she was completely unable to illustrate multiplication using paper and pencil, a number line, or her fingers. Similarly, she failed to solve problems that required an appreciation of some of multiplication's basic properties (e.g., if $12 \times 4 = 48$, then what is $4 \times 12$? If $12 \times 4 = 48$, then what is $12 + 12 + 12 + 12$?). Taken together, these patients exhibit a double dissociation between knowledge of memorized arithmetic facts and conceptual number knowledge, suggesting that the two types of number knowledge are functionally and anatomically distinct (Delazer & Benke, 1997; Hittmair-Delazer et al., 1994, 1995).

### *Dissociations between Exact and Approximate Calculation*

A related, but somewhat different, dissociation has been reported between exact calculation (e.g., determining the exact value of a multiplication problem by consulting memorized arithmetic facts) and approximate calculation (e.g., determining which of two Arabic numbers is larger, rejecting calculation results that are not even close to correct). For example, Dehaene and Cohen (1991) described a patient (NAU) who was severely impaired at exact calculation (e.g., he judged $2 + 2 = 5$ to be correct) but was relatively preserved at approximate calculation tasks, such as rejecting $2 + 2 = 9$ and determining which of two numbers is larger. Dehaene and Cohen (1997) described patients exhibiting both sides of the double dissociation; patient BOO suffered from a selective deficit of exact calculation with intact approximate calculation, while patient MAR exhibited preserved exact calculation with impaired approximate calculation. Behavioral, neuroimaging, and ERP results are also consistent with this dissociation (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999).

Dehaene and Cohen (1997) and Dehaene et al. (1999) interpreted these cases within their triple-code theory of number processing (Dehaene, 1992, 1997; Dehaene & Cohen, 1995). This theory distinguishes among three categories of number representations: (a) a visual Arabic code, which allows digit strings to be processed on a visuospatial sketchpad; (b) a verbal code, which is the primary code for accessing memorized arithmetic facts; and (c) an analogue magnitude code, which represents the quantity conveyed by numbers. In their view, exact calculation puts particular demands on access to memorized arithmetic facts that are represented in the verbal code, and damage to the verbal system therefore produces selective impairments in exact calculation (Cohen, Dehaene, Chochon, Lehericy, & Naccache, 2000; but for evidence that calculation skills can be preserved despite profound language impairments, see Rossor, Warrington, & Cipolotti, 1995). Operations such as single-digit multiplication, and to some extent addition, which tend to be solved by memory retrieval, are therefore predicted to be particularly vulnerable to damage of the verbal system. Conversely, approximate calculations put more emphasis on the magnitude of numbers rather than exact values and are therefore more dependent on the analogue magnitude system. Operations that are solved by accessing quantity representations rather than by direct memory retrieval (e.g., Arabic number comparison and even to some extent subtraction) are therefore predicted to be impaired when the analogue magnitude system is damaged.

### *Dissociations among Semantic Representations? Symbolic versus Analogue Codes*

In short, there is compelling evidence for two different, but related, dissociations in the number processing system: Conceptual number knowledge dissociates from arithmetic skills, and approximate calculation dissociates from exact calculation. Although these dissociations are different, both draw a distinction between representations that are assumed to be semantic (e.g., conceptual number knowledge, analogue magnitude information) and those that are more procedural (e.g., arithmetic skills, exact calculation). One potentially important difference is in the nature of the semantic representations that the two theoretical distinctions assume. As Delazer and Benke (1997) pointed out, the semantic representations that are assumed to underlie approximate calculation versus conceptual number knowledge seem quite different.

The triple-code theory assumes that approximate calculation depends on an analogue representation of number magnitudes. This magnitude representation is assumed to be something like a number line except that it becomes more compressed for higher numbers, like a logarithmic scale. It is used to represent the magnitude of numbers, and of the three codes in the model, it is the only one that is thought to be semantic. Proposals regarding conceptual number knowledge are not as explicit about the nature of the underlying representation, but descriptions of the knowledge itself suggest a representation that is more symbolic than could be supported by a representation of analogue magnitude. For example, it is hard to imagine how an analogue number line representation could represent the fact that multiplication corresponds to repeated addition.

The distinction between a symbolic/categorical code and an analogue code is a common theme in the study of human cognition. For example, drawing such a distinction has been important in explaining aspects of associative memory (Paivio's [1969, 1971] distinction between symbolic and analogue codes), mental imagery (Kosslyn's [1995] distinction between propositional and depictive codes), working memory (Baddeley's [1986] distinction between phonological and visuospatial codes), and spatial relations (Kosslyn et al.'s [1989] distinction between categorical and coordinate codes). In each case, the analogue code corresponds relatively directly to that which it represents (i.e., the representation and referent are analogous). Analogue codes are also continuous rather than discrete, and they cannot typically be precisely described verbally. By contrast, symbolic codes are assumed to consist of symbols whose meaning is defined arbitrarily; in particular, the representation is *not* assumed to resemble the referent directly. Symbolic codes are also discrete rather than continuous, and they are often assumed to be propositional or verbal (and hence easily verbalizable).

In this article, we investigate whether a similar distinction exists within the number processing system—between the representation of symbolic number knowledge, on the one hand, and that of analogue magnitude information, on the other. Specifically, are there distinct functional and neuroanatomical systems for processing symbolic number knowledge versus analogue magnitude information? Current evidence regarding this issue is mixed. Cipolotti, Butterworth, and Denes (1991) described a patient (CG) who could perform a variety of tasks involving the numerosities 1 through 4 but was unable to deal with higher numerosities. The impairment was apparent in some nonsymbolic magnitude tasks (e.g., comparing the numerosity of dot patterns, ordering dot patterns according to numerosity) as well as tasks that tapped higher level symbolic number knowledge (e.g., How many days in a week? How many eggs in a dozen?). This pattern of results is consistent with the hypothesis of a unitary system for dealing with symbolic number knowledge and magnitude information. Similarly, patient MAR (Dehaene & Cohen, 1997), whose ability to process magnitudes and approximate quantities was severely impaired, also exhibited deficits in answering questions that tapped symbolic number knowledge such as the number of days in a year.

On the other hand, some aspects of these patients' behavior suggest a possible dissociation between symbolic number knowledge and the ability to process analogue quantity information. For example, patient CG was still able to perform size judgment tasks (e.g., given pictures of familiar objects, choose the one that is biggest in real life) despite her profound deficit in number knowledge (Cipolotti et al., 1991). Similarly, patient MAR, who was severely impaired in tasks requiring an appreciation of magnitude, exhibited relatively preserved knowledge of famous numbers (504 = Peugeot, 1798 = French Revolution), of time, and of parity (Dehaene & Cohen, 1997). Delazer and Butterworth (1997) described a patient (SE) who was impaired on tasks requiring an appreciation of the cardinality of numbers (perhaps the most basic kind of symbolic number knowledge) but was able to compare the numerosity of dot arrays and sort them in series (a task that, as the authors noted, could be done without understanding the cardinality of numbers).

In this article, we describe a patient (MC) who exhibits a clear dissociation between these two types of semantic representations. MC displays a profound impairment in a variety of tasks that require symbolic number knowledge: naming numbers or other mathematical symbols, understanding rounding, dealing with clocks and time, and performing tasks that require an understanding of Arabic numbers. Nevertheless, her ability to represent and process analogue magnitude information is relatively preserved. To illustrate this dissociation between symbolic number knowledge and analogue magnitude information, we attempt to demonstrate three points in this article. First, MC is impaired on a wide range of tasks that involve symbolic number knowledge. Second, MC is not impaired on tasks that involve analogue magnitude information. Third, MC is not impaired on tasks that involve symbolic knowledge in other semantic categories (other than quantities). We begin by briefly describing the patient's history. We then present experimental tests that were designed to address each of the three points above. Many of the tasks were based on tests administered to patient CG (Cipolotti et al., 1991) that had been designed to demonstrate a category-specific semantic deficit in the number domain. Our findings are consistent with the hypothesis that symbolic number knowledge is a functionally independent component of the number processing system. They also suggest that symbolic number knowledge is category specific and that it is anatomically and functionally distinct from analogue magnitude representations.

## CASE HISTORY

MC is a 65-year-old right-handed female. She achieved a high school education and an electronics trade school degree in keypunch operation. Before she retired, MC worked at an electronics firm and worked with numbers and equations on a frequent basis. There is no reason to suspect an abnormal premorbid difficulty with numbers and mathematics. MC suffered a left hemisphere ischemic stroke on June 26, 1997. Magnetic resonance imaging revealed a small lesion in the subcortical white matter beneath the supramarginal and postcentral gyri of the left parietal lobe. The lesion extended far enough inferior to potentially involve white matter tracts of the superior temporal gyrus as well. Diffusion-weighted magnetic resonance imaging confirmed the acuity of the stroke (Fig. 1). A neurological examination was conducted on July 8, 1997, or 2 weeks after her initial stroke. She was only mildly aphasic, and her performance on all tests but the calculation test in the Neurobehavioral Cognitive Status Examination was in the normal to mildly impaired range. On the calculation test, MC was severely impaired to the point where she failed to answer any of the questions correctly. MC's language comprehension and production skills, both spoken and written, were only mildly impaired. On the Boston Diagnostic Aphasia Battery, her performance was in the normal range for all tests except the word dis-

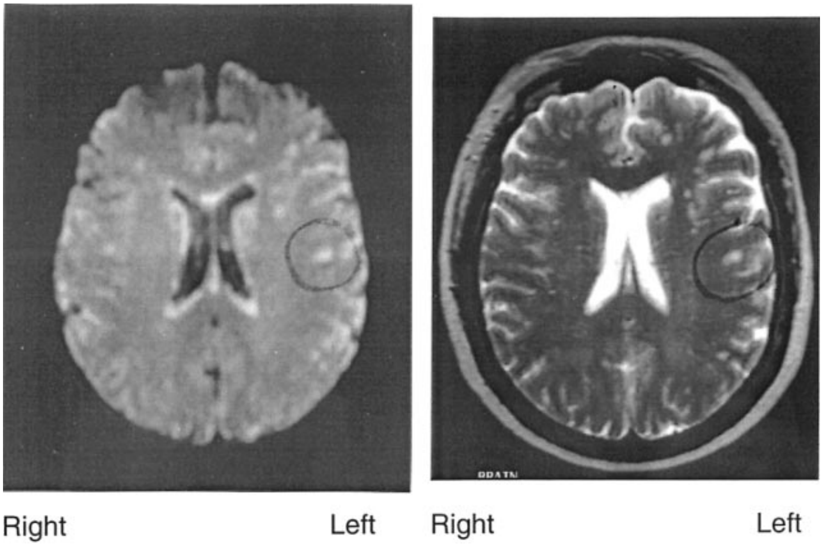


**FIG. 1.** Axial fluid attenuated inversion recovery (left) and axial T2-weighted (right) magnetic resonance imaging scans showing an area of increased signal consistent with an ischemic stroke in the subcortical white matter beneath the supramarginal and postcentral gyri of the left parietal lobe.

crimination test of the auditory comprehension section and the oral agility test of the oral expression section, both of which were mildly impaired. Aside from her number processing deficits, MC did not exhibit any of the other symptoms associated with Gerstmann's syndrome (e.g., left–right disorientation, finger agnosia, agraphia).

MC was tested on three occasions: July 9, 1997; May 4, 1998; and June 9, 1998. In all three sessions, MC was motivated, was cooperative, and worked hard to complete all of the tasks. MC's behavior was relatively stable across the testing sessions. Between the first and last testing sessions, MC received extensive therapy encouraging her to increment her way around the number symbols on an analogue clock, and she learned to map number names and symbols onto positions on the clock. Using this strategy, she was able to identify/name the numbers 0 to 12 (earlier she could recognize only 0 and 1). As a result of this therapy, she also developed the ability to count to 12 (earlier she could count only to 5). Aside from this modest recovery of function, however, few changes in her behavior were observed across the testing sessions. In particular, the dissociation between symbolic number knowledge and analogue magnitude processing was apparent in all testing sessions.

## SYMBOLIC NUMBER KNOWLEDGE

We begin by presenting a set of tests that illustrate that MC is impaired across a wide range of tasks involving symbolic number knowledge. First, we present tests that assess MC's basic arithmetic skills (e.g., simple calculations, counting). Then we describe tests designed to assess MC's semantic understanding of numbers and other kinds of symbolic number knowledge. The tests show that the impairment is not tied to a particular perceptual modality and suggest that MC's impairment extends to concepts that are closely related to Arabic numbers (e.g., non-numerical mathematical symbols, Roman numerals, rounding).

### Tests of Basic Arithmetic Skills

*Simple calculations in multiple formats and modalities of presentation.* MC was given a series of 20 cards with Arabic numbers or black dots on them along with calculation symbols that indicated a particular numerical operation, namely addition or subtraction, to be performed. She was asked to respond verbally. The numbers and dots represented single-digit numbers. For example, a card might show four dots on the left, a plus sign in the middle, and two dots on the right followed by an equal sign. The numerical version shows ''$4 + 2 =$ .'' The addition and subtraction problems were ones for which most people retrieve memorized facts rather than perform the calculation (McCloskey et al., 1991): $3 - 2$, $5 - 3$, $4 - 1$, $4 - 4$, $3 - 0$, $1 + 1$, $3 + 3$, $4 + 2$, $5 + 3$, and $4 + 1$. The same equations on the cards were then presented to MC verbally (e.g., ''What is 1 plus 1?'' ''What is 5 minus 3?''). MC was unable to perform any calculation regardless of its form or presentation. She scored 0/10 for number problems, 0/10 for dot problems, and 0/10 for verbally presented problems.

*Counting.* MC was asked to count forward from a given number. She was given 12, 1, 3, 30, and 22 as seed numbers. During the first testing session, MC could start at 1 and count to 5 but not beyond that. She was completely unable to count starting at 3, 12, 22, or 30. (As previously noted, she later developed the ability to count to 12.)

*Counting dots and beats.* MC was shown a card with black dots on it or listened (with eyes closed) to a set of auditory beats. The number of dots on a card or of beats heard for each trial was 1, 2, 3, 8, or 9. MC's task was to report the number of dots or beats without using her clock strategy. (Without this instruction, MC would use her fingers as place keepers and then use them to imagine incrementing around an analogue clock.) For the visual dots, MC was correct for the trials with 1, 2, and 3 dots but incorrect for the trials with 8 and 9 dots (3/5). For the auditory beats, MC was correct for the trials with 1 and 2 beats but incorrect for the other three (2/5).

### Tests of Semantic Understanding

*Number matching and naming.* We asked MC to discriminate and name three-dimensional Arabic numbers by touch as well as sight. In addition, we administered these visual and tactile tasks for three-dimensional letters. The letters and numbers were made of raised rubber cutouts from a child's alphabet puzzle set. MC was presented with an uppercase letter on her left and was asked to determine which of two choices of lowercase letters on her right matched it. She was also asked to determine which of two number choices matched a target number. At the end of each trial, MC was asked to name the target number or letter. She performed this task both by sight and by touch (with eyes closed).

MC was 100% accurate in matching visually and tactually presented numbers and letters. She could also name the letters accurately but exhibited a category-specific deficit in naming numbers. In the visual conditions, she was able to match 7/7 letters and 7/7 numbers. She was also able to name 7/7 letters but was able to name only 2/7 numbers (0 and 1). In the tactile conditions, MC matched 7/7 letters and 7/7 numbers. Although she was able to name 7/7 letters using touch alone, she was able to name only 2/7 numbers (again, the numbers 0 and 1). Her problem with naming appears to have been due to a problem in recognition rather than a word-finding difficulty. She claimed no knowledge of the numbers she failed to name, and she did succeed in naming the 2 she claimed to recognize (0 and 1). However, even for these 2 numbers, she did not behave normally; she identified 0 by first noting that it was a circle (she had learned that a circle was a 0). Similarly, she identified 1 by first recognizing it as a straight vertical line.

*Writing numbers to dictation.* MC was asked to write 10 Arabic numbers from spoken dictation. This test was administered after MC had developed some ability to deal the numbers 1 to 12 by using an analogue clock, and her performance reflected this; MC correctly wrote down all 6 single-digit numbers but failed on all 4 of the multidigit numbers (15, 512, 444, and 987) (6/10).

*Number ordering: What comes before or after?* MC was first provided with 1 number and then asked what number came *before* that. She was given 10 different seed numbers. This test was also administered after MC had recovered some ability to deal with the numbers 1 to 12 and a similar pattern was observed: She was correct for all 7 numbers between 2 and 12 but missed the other 3 (7/10). In response to 1, she said, ‘‘No number comes before 1’’ (despite the fact that she could recognize and name 0). She also missed 15 and 20. For 15, she answered incorrectly and then remembered her strategy to cover the 1 with her finger and reasoned, ‘‘If there is a 5, then a 4 comes before it and you then put in a 1.’’ Nonetheless, she did not know that the number was 14. For the number 20, she guessed 2.

Second, she was provided with a number and asked what number came *after* that. Again, she was given 10 different seed numbers. Again, she demonstrated correct performance on numbers between 1 and 12 and missed double-digit numbers (e.g., 15, 20) (8/10).

*Recognition of mathematical symbols.* MC was shown individual mathematical symbols on index cards (+, −, =, %, decimal point, and ÷) and was asked to name them and tell the experimenter what they meant or how they were used. Although she could name some mathematical symbols (+, −, and =), she could not recognize others (%, decimal point, and ÷). She was also unable to indicate how the symbols she named are used.

*Roman numerals and numbers with decimal points.* MC was tested to see whether she could identify Roman numerals and numbers with decimal points. She failed to identify any of the 10 numbers we presented. She said that she could recognize Roman numerals before her stroke. When presented with numbers with decimal points, she said that she did not know ‘‘what that dot is.’’

*Rounding.* Similarly, MC was completely unable to round numbers. When asked to do so, she complained that she did not understand what rounding meant.

*1’s, 10’s, 100’s columns.* MC was shown 10 multiple-digit numbers and was asked to indicate which digits represented the 1’s, 10’s, 100’s, and 1000’s columns of numbers. MC said that she did not understand the question. She could indicate which digit was farthest to the left or right but not what the relative position of a digit among other digits meant.

*Cardinal facts: Knowledge of units.* MC was asked the following 10 questions regarding how many units are in a particular measure. How many hours are in a day? How many halves in a football game? How many days in a year? How many ounces in a cup? How many feet in a yard? How many eggs in a dozen? How many inches in a foot? How many centimeters in a meter? How many people on a baseball team? How many days in a week? MC was able to answer only 1 of the questions and answered that one only partially; she counted off the days of the week on her fingers and responded ‘‘the number after 6.’’

*Reading the clock.* MC was shown an analogue clock on which the hands were arranged to indicate various times. Two of the times were on the hour, two were partial hour times, and two were on the quarter and half-hour. This test was administered after MC had recovered some ability to deal with the numbers 1 to 12, and she was indeed able to read the times that were on the hour (3 o’clock and 6 o’clock).

Thus, she was accurate for 2/6 trials. When the indicated times were not on the hour, MC read the two numbers separately but was unable to combine them. For example, she reported ''4'' and ''9'' when the clock indicated 4:45, ''2'' and ''5'' when the clock indicated 5:10, ''7'' and ''3'' for 7:15, and ''10'' and ''6'' for 10:30. When questioned, she said that she did not know what a half-hour was or how many minutes made up an hour.

*Time calculations.* To determine whether MC could use numbers related to time, we asked her several practical questions that involved time calculation in her everyday life. If it is now 3:00 p.m. and your meal comes at 5:00 p.m., then how long will you have to wait for dinner? You have just taken a pain relief pill that lasts for 3 h. If you watch a 2-h movie, will you have to take another pill immediately when it is over? If you have physical therapy until 10:00 a.m. and speech therapy starts at 10:30 a.m., then how long a break do you have between them? You have a 15-min doctor's appointment at 11:00 a.m. Assuming that your doctor is on time and that your appointment takes the full 15 min, how long do you have to eat lunch before your next 12:00 p.m. appointment? MC claimed that she no longer understood times and was unable to answer any of the questions (0/4 correct). For example, when asked how long she had to wait for dinner, she replied that dinner just comes when it comes in the hospital.

*Reading and defining number-related words.* MC was asked to read and define number concept words. She was told to give her best guesses. Her responses are shown in Table 1. MC demonstrated perfect performance reading the words (13/13), and she clearly still understood the non-numerical semantics of the words (e.g., quarter: ''coin''; quartet: ''music guys''), but in only one case did her definition demonstrate a clear understanding of the numerical information conveyed by the term (twin: ''two kids'').

## *Discussion*

MC had difficulty with every task we administered that involved symbolic number knowledge. In addition to being impaired on tasks that required some semantic understanding, MC was impaired on basic arithmetic tasks such as fact retrieval and counting. Therefore, she does not exhibit a dissociation between skills and understanding, a point we return to in the General Discussion. Her knowledge of basic mathematical concepts (e.g., rounding, 1's vs 10's columns in multidigit numbers), and even of

TABLE 1
Definitions of Quantity Words

| Word | Response |
|---|---|
| Dozen | No idea, no guess |
| Half | Half a gallon |
| Score | Keep score when bowling |
| Pair | Pair of nylons |
| Gross | Yucky |
| Single | I'm single and available |
| Trio | No guess |
| Quartet | Music guys |
| Twin | Had kids, 2 kids |
| Unit | Stereo unit |
| Quarter | Coin |
| Sextet | No guess |
| Fifth | Grade 5 |

basic declarative facts involving numbers (e.g., How many hours are in a day? What does quartet mean?), is severely impaired. Her deficit is clearly not a peripheral perceptual impairment and is not tied to any single perceptual modality. Her difficulty in answering simple questions that require numbers as answers and the sparseness of numerical information in her definitions are particularly compelling. Both suggest a loss of symbolic number knowledge.

## APPROXIMATE CALCULATION AND PROCESSING OF MAGNITUDE INFORMATION

Next, we present a set of tests designed to assess MC's ability to process analogue magnitude information. We begin with two tasks that have previously been used to tap approximate, as opposed to exact, calculation abilities and that are assumed to require the processing of analogue quantity information: Arabic number magnitude comparison and the thermometer task (mark the appropriate placement of an Arabic number on a thermometer) (Dehaene & Cohen, 1991, 1997; Dehaene et al., 1999). In addition to magnitude processing, however, both of these tasks also require the translation of Arabic numbers (a symbolic representation) into a magnitude representation. Consistent with MC's impairment in symbolic number processing, she exhibits deficits on these tasks. However, when we assess MC's ability to perform tasks that require magnitude processing but do not require the translation of number symbols, her performance is completely intact.

### *Tests Involving Arabic Numbers*

*Arabic number comparison.* MC was presented with 20 number pairs (10 single digit and 10 double digit). Her task was to identify which of the numbers was larger. Using a laborious counting strategy (by the time of this testing, MC had recovered the ability to count to 12 and to name the digits up to 12 as a result of her therapy using clocks), she correctly chose the larger number for all of the single-digit numbers but said that she could not perform the task for the double-digit number pairs (10/20). When asked to try, she again tried to apply a counting strategy to one of the two columns in the double-digit numbers. This worked in 7 of the cases (e.g., when the 10's column differed and she applied it to the 10's column) but not in the other 3 (e.g., when she applied it to the 1's column instead of the 10's column).

*Thermometer test.* MC was shown 10 line drawings of a thermometer with 0 labeled at the bottom and 100 labeled at the top (with no marks in between) and was asked to mark the spot on the thermometer where a particular number should be placed. The numbers ranged from 1 to 99. Her performance is shown in Fig. 2. Clearly, MC displayed very poor absolute placement of the numbers. Her placement of the numbers relative to each other was better, indicating that some magnitude information may have been getting processed. However, MC's behavior was quite abnormal and relied heavily on counting on her fingers (again, by the time of this testing, MC had recovered the ability to count to 12). For the double-digit numbers, MC used her fingers to cover up one digit and then counted on her fingers to determine spatially the magnitude of the remaining number. She also said that she had learned that two numbers together (i.e., double-digit numbers) are bigger than one number alone (i.e., single-digit numbers). In short, MC was dramatically impaired on this task.

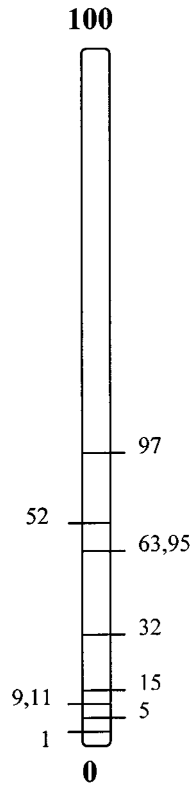


**FIG. 2.** The placement of individual numbers on a scale of 0 to 100 (thermometer task) by patient MC.

## *Tests without Arabic Numbers*

*Size judgments.* MC was shown 60 pairs of pictures taken from the Snodgrass and Vanderwart (1980) collection. The objects in the pairs were scaled so that the pictures themselves were the same size. These pairs were separated into categories: fruits and vegetables, animals, household items, musical instruments, and vehicles. MC was asked to determine which object in each pair was larger in real life. She correctly answered all 60 questions.[1]

*Size ordering task.* MC was given two sets of five objects from the same pictures as above. For each set, she was asked to order them according to how large they are in real life. MC correctly ordered all of the objects in terms of their size.

*Comparing perceptual quantities.* MC was shown two piles of pennies or pictures of two glasses of water and was asked to point to the pile/glass that had more pennies/water. MC was 100% accurate (5/5).

*Comparing estimated quantities.* MC was asked to answer 10 questions that required comparing the magnitude of imagined quantities. For example, she was asked questions such as ''Would there be more coffee beans or sugar grains in a cup?'' MC answered all of these questions quickly and correctly.

[1] People are faster to compare the sizes of object pairs that differ substantially in size relative to object pairs whose sizes are similar (Kerst & Howard, 1977; Moyer, 1973; Moyer & Bayer, 1976). This effect is typically interpreted as evidence that people use some kind of analogue quantity representation in making these comparisons.

*Numerosity: Which is more?* MC was shown two sets of irregularly sized and placed dots and was asked which set contained more dots. Each set contained between two and five dots. The stimuli were designed so that she could not use spatial position or physical space covered to answer the questions. She was also asked to perform three Piagetian tasks in which she had to indicate which row of linearly organized dots (2–5 dots per row) with different amounts of space between them had more dots. She displayed none of the errors prominent in the performance of young children; size and spatial arrangements of items did not fool her. She correctly answered 5/5 dot problems and 3/3 of the Piagetian tasks. She did not try to count and did not require any special strategies to complete these tasks.

*Numerosity: Ordering quantities.* MC was given 10 index cards with one to six irregularly sized and placed dots on them. She was asked to put the cards in order from left to right, placing the cards with the fewest dots farthest on the left and progressively placing cards with more dots farther to the right. She was also instructed to put cards with the same number of dots in the same position. She had no difficulty performing this task (10/10) and again did not count.

*Frequency estimation.* MC was read lists of 20 words and was asked to judge the relative frequency of category exemplars (without explicitly counting the exemplars). There were two versions of the test, with two lists for each version. In the first and easier version, she was told the names of the two categories before hearing the list items: ‘‘You will hear 20 words. Some are names of musical instruments, and some are names of vegetables. At the end, I want you to tell me whether the list had more instruments or vegetables. Don't count [MC gave a big sigh of relief!]; just get a feel for which occurs more often.’’ There were three times as many exemplars for the more frequent category in the easy versions (12 instruments vs 4 vegetables [along with four fillers], 12 clothing items vs 4 flowers [along with four fillers]).

The more difficult version of the task did not specify the categories ahead of time; she was told to listen to the list of items, and then she would be asked some questions about it. At the end of the list, she was told that some of the items had been ‘‘category 1’’ and some had been ‘‘category 2’’ and was asked which category had been more frequent. The more difficult version also involved a more difficult discrimination, as the ratio of the relevant frequencies was less than 2 (7 fruits vs 4 furniture items [along with 9 fillers]; 7 body parts vs 4 animals [along with 9 fillers]). Filler items occupied the first and last position on the list. The last relevant item on a list was from the most frequent category for one list of each version and from the least frequent category for the other list.

MC performed both the easy and the difficult relative frequency judgments with ease and correctly judged the more frequent category in all cases.

### *Discussion*

MC is able to process magnitude information so long as the task does not require symbolic number knowledge. She can deal with continuous analogue quantities (e.g., size) despite her difficulty with tasks that require symbolic knowledge about numbers and related concepts. By contrast, any task that requires an understanding of Arabic numbers or number words is difficult for MC. For example, in tasks that tap approximate calculation but require the translation of Arabic numbers into magnitude representations (including number comparison and the thermometer task), MC is impaired. Although her performance suggested that some magnitude information might be getting through, her strategy and performance were very abnormal. It is only when an understanding of number symbols is *not* required (e.g., frequency estimations, comparing the numerosity of dot patterns) that MC performs well. Even when the stimuli were discrete (e.g., dots, category exemplars), MC did not count but rather appeared to adopt a more analogue approximate representation of the quantity of dots or the frequency of category exemplars.

## PRESERVED PROCESSING OF OTHER SEMANTIC CATEGORIES

Finally, we present a set of tests designed to assess MC's ability to process information from semantic categories that are not directly related to numbers. The processing of non-numeric semantic categories in patients with conceptual number deficits has rarely been systematically tested, but the current evidence favors category specificity. For example, patient CG, who exhibited a profound deficit in conceptual number knowledge for numerosities above 4, was relatively normal on a number of tasks tapping semantic knowledge of other categories (e.g., verbal fluency, picture naming, synonyms) (Cipolotti et al., 1991). She did, however, exhibit some problems with word definitions (20%–30% errors) and was quite impaired on tests of non-numeric ordinal structure (e.g., the alphabet, days of the week, months of the year). This patient also exhibited other behavioral deficits in addition to her problems with numbers (e.g., spatial deficits, body schema deficits, other impairments associated with Gerstmann's syndrome). It is therefore possible that an impairment in some more general cognitive system (e.g., spatial processing) played a role in some of her number processing problems. By contrast, MC did not exhibit any impairments in tasks tapping other semantic categories, including tests of ordinal structure, and she also did not suffer from any other problems associated with Gerstmann's syndrome.

### *Tests*

*Object discrimination for different semantic categories.* MC was able to point to 60/60 pictures of items from seven different semantic categories (animals, fruits, vegetables, body parts, musical instruments, vehicles, and household objects).

*Naming common objects.* MC named 19/20 line drawings of common objects selected from the Snodgrass and Vanderwart (1980) collection; her 1 error was mistaking an apple for a cherry. She also named 60/60 pictures of items from seven different semantic categories.

*Generation of non-numerical sequences (ordinal information).* MC could correctly recite the days in a week (7), the months of the year (12), and the alphabet (26). By contrast, at the time this test was administered, she was able to count only to 5 and was completely unable to count forward by 5's and 10's.

*Event ordering.* MC answered quickly and correctly 15 questions about what item followed a given day, month, or letter.

*Temporal ordering.* MC was given five pairs of famous people names. She was asked to indicate who was born first (e.g., Abraham Lincoln or George Washington). She answered all five questions correctly.

*Linear ordering.* MC was given five linear ordering problems (e.g., If Bill is taller than John, and John is taller than Susie, then who is shortest?). MC answered 4/5 correctly.

*Generation of words from designated semantic categories.* In a verbal fluency task, MC was asked to name as many fruits as possible in 60 s. Similarly, she was asked to name as many vegetables and as many animals as possible. She produced an average of 12 items per category, which is within the normal range.

*Defining concrete and abstract words of different frequency.* MC was asked to define 10 high-frequency concrete words, 10 low-frequency concrete words, 5 high-frequency abstract words, and 5 low-frequency abstract words. MC correctly defined all of the words.

*Identifying synonyms.* MC correctly identified all of the pairs of synonyms from a list of 48 word pairs.

### *Discussion*

In contrast to her dramatic impairments on tasks involving symbolic number knowledge, MC exhibited normal performance with tasks involving other semantic categories. She recognized and named items from a variety of non-numerical categories, she correctly recited non-numerical sequences, she made judgments based on sequential order, and she easily retrieved and processed the semantics of all the non-numerical words we tested. In many cases, MC exhibited dramatic impairments on these very same tasks when they involved number concepts. Based on these results, it appears that MC's deficit is category specific; that is, it affects numbers and related concepts but not other semantic categories.

## GENERAL DISCUSSION

Based on MC's pattern of impaired and preserved performance, we would characterize her deficit as an impairment in symbolic number knowledge. Table 2 presents a summary of the results of the experimental tests. Her ability to recognize, understand, define, retrieve, and manipulate symbolic number information is severely compromised, while other cognitive functions are relatively well preserved. The impairment is multimodal, being manifest whether the tests are visual, auditory, or tactile. It appears to be a deficit in semantic knowledge, but it extends to arithmetic skills as well. In contrast to her severe deficit in performing tasks that involve symbolic number knowledge, MC shows relatively normal performance in processing nonsymbolic analogue quantities. Finally, MC's impairments are specific to numbers and related concepts and do not extend to any other semantic domains that we have tested.

### *Symbolic Number Knowledge versus Analogue Magnitude Information*

Previous patients exhibiting semantic number deficits have typically been characterized either as being impaired in approximate calculation (Dehaene & Cohen, 1997; Dehaene et al., 1999) or as being impaired in conceptual number knowledge (Cipolotti et al., 1991; Delazer & Benke, 1997; Warrington, 1982; Weddell & Davidoff, 1991). The relationship between these two characterizations, however, is unclear. Do approximate calculation and conceptual number knowledge depend on a unitary component of the number processing architecture, or do analogue magnitude information and symbolic number knowledge depend on partially distinct systems? MC's case makes clear that the two types of semantic information can be dissociated. Despite her profound impairment in symbolic number knowledge, her ability to process analogue magnitude information is intact.

Current models of the functional architecture of number processing do not yet distinguish between these two types of semantic number information. For example, the modular model of number processing (McCloskey, 1992; McCloskey et al., 1985) assumes a unitary abstract representation of number meaning that is assumed to underlie all arithmetic operations and number transcoding tasks. Although the triple-code theory proposes that different operations depend on different numeric codes

TABLE 2
Summary of MC's Performance across a Set of Experimental Tasks

| Experimental task | Performance |
|---|---|
| Tasks requiring symbolic number knowledge | |
| Simple calculations | Impaired (0/9 Arabic numbers, 0/9 dots, 0/9 verbal) |
| Counting | Impaired (1–5 but not beyond, only from 1) |
| Counting dots and beats | Impaired (3/5 dots, 2/5 beats) |
| Number naming | Impaired (2/7 both by vision and touch) |
| Writing numbers to dictation | Impaired (6/10 but only below 12) |
| Before/after numbers | Impaired (15/20 but only below 12) |
| Mathematical symbols | Impaired (named 3/6, defined 0/6) |
| Roman numerals | Impaired (0/10) |
| Rounding | Impaired (0/10) |
| 1's, 10's, and 100's columns | Impaired (0/10) |
| Cardinal facts | Impaired (1/10) |
| Reading a clock | Impaired (2/6) |
| Time calculations | Impaired (0/4) |
| Defining number-related words | Impaired (see Table 1) |
| Magnitude tasks involving arabic numbers | |
| Arabic number comparison | Impaired (10/20 with abnormal strategy) |
| Thermometer test | Impaired (see Fig. 2) |
| Magnitude tasks without arabic numbers | |
| Size judgments | Normal range (60/60) |
| Size ordering | Normal range (10/10) |
| Comparing perceptual quantities | Normal range (5/5) |
| Comparing estimated quantities | Normal range (10/10) |
| Dot numerosity comparisons | Normal range (8/8) |
| Ordering dot numerosities | Normal range (10/10) |
| Frequency comparison | Normal range (4/4) |
| Tasks requiring non-numeric symbolic knowledge | |
| Pointing to pictures | Normal range (60/60) |
| Naming common objects | Normal range (19/20) |
| Non-numeric sequences: Generation | Normal range (weekdays, alphabet, months) |
| Non-numeric sequences: Next item | Normal range (15/15) |
| Temporal ordering | Normal range (5/5) |
| Linear syllogisms | Normal range? (4/5) |
| Verbal fluency | Normal range (12 items/min) |
| Word definitions | Normal range (30/30) |
| Synonym identification | Normal range (48/48) |

(e.g., a verbal code for memorized multiplication problems, a magnitude code for Arabic number comparison), only one of these codes (the analogue magnitude code) is assumed to represent semantic information about numbers (Dehaene, 1992; Dehaene & Cohen, 1991, 1995, 1997). MC's case suggests that a distinction should be drawn between symbolic number knowledge and analogue magnitude information.

This dissociation between symbolic and analogue information also suggests that a distinction should be drawn between different kinds of approximation/magnitude tasks. In some of these tasks, the analogue magnitude information is reflected explicitly in the stimuli themselves, whereas in others, a number symbol must first be translated into a magnitude code. For example, when comparing the amount of water in two glasses, the magnitude information is reflected explicitly in the stimulus, and there is no need to translate from an arbitrary symbol (e.g., a number) into the magnitude code. Similarly, when comparing the numerosity of dots patterns or the frequency of occurrence of different events, the translation of an arbitrary symbol into the magnitude code is not required. Other approximation/magnitude tasks, however, do require such a translation (in addition to the ability to represent and manipulate magnitudes). For example, comparing the magnitude of two Arabic numbers clearly requires translating the number symbols into a magnitude representation. Similarly, marking the appropriate location of an Arabic number on a magnitude scale (e.g., mark the approximate location of 82 on a line that runs from 0 to 100) also requires translating the Arabic number into a magnitude representation. These kinds of tasks are prototypical approximate calculation tasks and have been shown to dissociate from exact calculation tasks (Dehaene & Cohen, 1991, 1997). Nevertheless, we propose that such tasks do require some symbolic number knowledge and should therefore dissociate from more pure magnitude tasks that do not require the translation of number symbols. Consistent with this assumption, MC is more impaired on such tasks than on the pure magnitude tests described previously, although there is some evidence that some magnitude information can still be used.

The distinction between analogue magnitude information and symbolic number knowledge may reflect a distinction between innate and learned representations in the number system (Dehaene, 1997). Dehaene, Dehaene-Lambertz, and Cohen (1998) presented evidence that preverbal infants, as well as many nonhuman animals, possess a biologically determined, domain-specific representation that can be used to perceive, discriminate, and manipulate small quantities. For example, if infants are shown a sequence of dot patterns, all of which have the same numerosity (say, 2), and are then shown a pattern with a different numerosity (say, 3), they will look at the new pattern longer than they will look at a pattern in which the numerosity was not changed (Starkey & Cooper, 1980). Similarly, infants are able to discriminate different numbers of syllables (Bijeljac-Babic, Bertoncini, & Mehler, 1993) and visual actions (Wynn, 1996). Nonhuman animals have been shown to possess similar abilities (Davis & Perusse, 1988; Gallistel & Gelman, 1992). These abilities are typically assumed to depend on an analogue magnitude representation like that hypothesized to be intact in patient MC (Dehaene et al., 1998; Gallistel & Gelman, 1992). By contrast, knowledge of Arabic numbers and other types of symbolic number knowledge is not innate and can be acquired only through extensive training. And tasks that depend on symbolic number knowledge, like those impaired in MC, cannot be spontaneously performed by preverbal infants or nonhuman animals (although nonhuman animals have been shown to be able to perform some tasks involving Arabic numbers [Matsuzawa, 1985; Washburn & Rumbaugh, 1991], such behavior requires extensive training). In short, evidence suggests that the ability to represent and manipulate analogue magnitude information is genetically predetermined, whereas symbolic number knowledge requires systematic training to develop. MC's dissociation between symbolic number knowledge and analogue magnitude information may therefore reflect a dissociation between learned number representations (symbolic number knowledge) and number representations that are genetically predetermined (analogue magnitude information).

### *Relationship to Previous Neuropsychological Work*

Of the previously reported cases of acalculia, MC is probably most similar to patient CG (Cipolotti et al., 1991). CG also exhibited a selective impairment in tasks requiring number knowledge, the impairment was multimodal, and the category of numbers was disproportionately impaired. An NMR scan on CG revealed an area of left fronto-parietal hypodensity. This lesion was more extensive than MC's but may well have included areas damaged in MC.

There are a number of important differences, however, between MC and CG. For example, CG exhibited impairments on some magnitude tasks on which MC was preserved. She had difficulty comparing the numerosity of dot patterns and sorting dot patterns according to numerosity. CG was also impaired on word definitions and tests of non-numeric ordinal structure (e.g., days of the week, alphabet), tasks on which MC performed well. Cipolotti et al. (1991) distinguished ways in which CG's problems with non-numeric sequences were different from her problems with numbers. MC's case demonstrates that the two deficits can indeed be dissociated. Furthermore, CG exhibited the classical signs of Gerstmann's syndrome—finger agnosia, left–right disorientation, and agraphia—in addition to her acalculia, and she was also profoundly alexic. MC did not exhibit any of these other problems (presumably due to her more restricted lesion), and her case therefore makes clear that these other neurological deficits can be dissociated from the symbolic number knowledge impairment.

It is also worth discussing MC's case in light of the dissociation between conceptual knowledge and arithmetic skills. Although MC shows a clear conceptual deficit, she is also impaired in basic arithmetic skills (e.g., counting, arithmetic fact retrieval) that are known to dissociate from conceptual deficits (Delazer & Benke, 1997; Hittmair-Delazer et al., 1994, 1995). The term *symbolic number knowledge* is therefore meant to include, but not be limited to, conceptual number knowledge. Similarly, symbolic number knowledge is *not* meant to refer exclusively to a knowledge of number symbols. Rather, it is meant to refer to any kind of symbolic knowledge related to numbers (e.g., knowledge of rounding, of what a quartet is, and of what a decimal point means).

Perhaps MC's behavior reflects two separate and independent underlying deficits: one in practiced skills and another in conceptual knowledge. One problem with this interpretation is that both deficits appear to be specific to the category of numbers. For example, she was not impaired in reciting days of the week or months of the year (practiced skills) and was also able to define concepts from a variety of non-numeric semantic categories (conceptual knowledge). One would therefore need to assume that two separate lesions both affected number information selectively, one impairing symbolic number knowledge and another impairing practiced number skills. The idea that two independent lesions would happen to impair exactly the same category of information is obviously exceedingly unlikely. A more parsimonious interpretation is that even practiced number skills make some demands on symbolic number knowledge and that when the system underlying that knowledge is extensively damaged, it can affect practiced number skills as well.

MC's case also provides perhaps the strongest evidence to date that symbolic knowledge about numbers is dissociable from symbolic knowledge about other semantic categories. In keeping with previous studies suggesting that impairments in number knowledge (Cipolotti et al., 1991) can doubly dissociate from impairments in knowledge of other semantic categories (Thioux et al., 1998), MC's number impairments did not extend to any other semantic categories that we tested. The dissociation in MC was particularly clear in that her deficit did not even affect ordinal structures in other domains (e.g., alphabet, days of the week) and was present despite the lack of any spatial deficits or other symptoms of Gerstmann's syndrome. Her case therefore strongly suggests that number knowledge itself can indeed be selectively impaired, independent of other more general cognitive systems on which number processing might rely. This interpretation is consistent with the more general hypothesis that semantic memory includes a component devoted to numbers. Whether this component responds to the number category itself or to some feature that is typically associated with numbers (e.g., their abstractness) remains to be tested.

Finally, MC's case has implications for hypotheses regarding number processing in the right hemisphere. There is evidence that the right hemisphere is capable of a fair amount of number processing, including recognizing Arabic numbers and comparing their magnitude (Dehaene & Cohen, 1995). For example, these tasks are often preserved in patients with extensive left hemisphere damage (Dehaene & Cohen, 1991; Grafman, Kampen, Rosenberg, Salazar, & Boller, 1989) and in the right hemisphere of split-brain patients (Cohen & Dehaene, 1996; Seymour, Reuter-Lorenz, & Gazzaniga, 1994). In keeping with these findings, the triple-code theory of number processing assumes that the right hemisphere is capable of a variety of tasks that do not require use of the verbal code such as recognizing and comparing Arabic numbers (Dehaene & Cohen, 1995). Nevertheless, MC exhibited impairments on such tasks despite the fact that her lesion was strictly lateralized to the left hemisphere. Patient CG, who also suffered from a left lateralized lesion, also exhibited impairments on these tasks (Cipolotti et al., 1991). These discrepancies raise the possibility that the number processing capabilities of the right hemisphere differ across individuals and that the triple-code theory's assumption of right hemisphere number processing is correct only in some individuals. Individual differences in performance on these tasks could result from reorganization following damage or simply reflect variability that is present in the normal population. An alternative possibility is that the left hemisphere partially inhibits number processing in the right hemisphere and that significant left hemisphere damage (or commisurotomy) therefore disinhibits number processing in the right hemisphere. According to this interpretation, the smaller left hemisphere lesions in MC and CG may have spared the inhibition of right hemisphere number processing, thereby undermining the right hemisphere's ability to display its real number processing abilities. Such an interpretation could potentially reconcile MC and CG's impairments with the triple-code theory's assumption of right hemisphere number processing.

To summarize, the major conclusion that we draw based on patient MC's performance is that there is a dissociation between symbolic number knowledge and analogue magnitude information. This dissociation has important implications for the functional organization of number processing in the brain but is not yet reflected in current theories of number processing.

## REFERENCES

Baddeley, A. D. (1986). *Working memory.* New York: Oxford Univ. Press.

Bijeljac-Babic, R., Bertoncini, J., & Mehler, J. (1993). How do 4-day-old infants categorize multisyllabic utterances? *Developmental Psychology,* **29,** 711–721.

Cipolotti, L., Butterworth, B., & Denes, G. (1991). A specific deficit for numbers in a case of dense acalculia. *Brain,* **114,** 2619–2637.

Cohen, L., & Dehaene, S. (1996). Cerebral networks for number processing: Evidence from a case of posterior callosal lesion. *Neurocase,* **2,** 155–173.

Cohen, L., Dehaene, S., Chochon, F., Lehericy, S., & Naccache, L. (2000). Language and calculation within the parietal lobe: A combined cognitive, anatomical, and fMRI study. *Neuropsychologia,* **38,** 1426–1440.

Davis, H., & Perusse, R. (1988). Numerical competence in animals: Definitional issues, current evidence, and a new research agenda. *Behavioral and Brain Sciences,* **11,** 561–579.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition,* **44,** 1–42.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics.* New York: Oxford Univ. Press.

Dehaene, S., & Cohen, L. (1991). Two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia,* **29,** 1045–1074.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition,* **1,** 83–120.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex,* **33,** 219–250.

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neuroscience,* **21,** 355–361.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science,* **284,** 970–974.

Delazer, M., & Benke, T. (1997). Arithmetic facts without meaning. *Cortex,* **33,** 697–710.

Delazer, M., & Butterworth, B. (1997). A dissociation of number meanings. *Cognitive Neuropsychology,* **14,** 613–636.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition,* **44,** 43–74.

Grafman, J., Kampen, D., Rosenberg, J., Salazar, A., & Boller, F. (1989). Calculation abilities in a patient with a virtual left hemispherectomy. *Behavioral Neurology,* **2,** 183–194.

Hittmair-Delazer, M., Sailer, U., & Benke, T. (1995). Impaired arithmetic facts but intact conceptual knowledge: A single-case study of dyscalculia. *Cortex,* **31,** 139–147.

Hittmair-Delazer, M., Semenza, C., & Denes, G. (1994). Concepts and facts in calculation. *Brain,* **117,** 715–728.

Kerst, S. M., & Howard, J. H. (1977). Mental comparisons for ordered information on abstract and concrete dimensions. *Memory & Cognition,* **5,** 227–234.

Kosslyn, S. M. (1995). Mental imagery. In S. M. Kosslyn & D. N. Osherson (Eds.), *An invitation to cognitive science: Visual cognition* (2nd ed., Vol. 2, pp. 267–296). Cambridge, MA: MIT Press.

Kosslyn, S. M., Koenig, O., Barrett, A., Cave, C. B., Tang, J., & Gabrieli, J. D. E. (1989). Evidence for two types of spatial representations: Hemispheric specialization for categorical and coordinate relations. *Journal of Experimental Psychology: Human Perception and Performance,* **15,** 723–735.

Matsuzawa, T. (1985). Use of numbers by a chimpanzee. *Nature,* **315,** 57–59.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition,* **44,** 107–157.

McCloskey, M., Aliminosa, D., & Sokol, S. M. (1991). Facts, rules, and procedures in normal calculation: Evidence from multiple single-patient studies of impaired arithmetic fact retrieval. *Brain and Cognition,* **17,** 154–203.

McCloskey, M., Caramazza, A., & Basili, A. (1985). Cognitive mechanisms in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition,* **4,** 171–196.

Moyer, R. S. (1973). Comparing objects in memory: Evidence suggesting an internal psychophysics. *Perception & Psychophysics,* **13,** 180–184.

Moyer, R. S., & Bayer, R. H. (1976). Mental comparison and the symbolic distance effect. *Cognitive Psychology,* **8,** 228–246.

Paivio, A. (1969). Mental imagery in associative learning and memory. *Psychological Review,* **76,** 241–263.

Paivio, A. (1971). *Imagery and verbal processes.* New York: Holt, Rinehart & Winston.

Rossor, M. N., Warrington, E. K., & Cipolotti, L. (1995). The isolation of calculation skills. *Journal of Neurology,* **242,** 78–81.

Seymour, S. E., Reuter-Lorenz, P. A., & Gazzaniga, M. S. (1994). The disconnection syndrome: Basic findings reaffirmed. *Brain,* **117,** 105–115.

Snodgrass, J. G., & Vanderwart, M. (1980). Standardized set of 260 pictures: Norms for name agreement, image agreement, familiarity, and visual complexity. *Journal of Experimental Psychology: Human Learning and Memory,* **6,** 174–215.

Sokol, S. M., McCloskey, M., & Cohen, N. J. (1989). Cognitive representations of arithmetic knowledge: Evidence from acquired dyscalculia. In A. F. Bennett & K. M. McConkey (Eds.), *Cognition in individual and social contexts* (pp. 577–591). Amsterdam: North Holland.

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science,* **210,** 1033–1035.

Thioux, M., Pillon, A., Samson, D., de Partz, M. P., Noel, M. P., & Seron, X. (1998). The isolation of numerals at the semantic level. *Neurocase,* **4,** 371–389.

Warrington, E. K. (1982). The fractionation of arithmetical skills: A single case study. *Quarterly Journal of Experimental Psychology Section A: Human Experimental Psychology,* **34,** 31–51.

Washburn, D. A., & Rumbaugh, D. M. (1991). Ordinal judgments of numerical symbols by macaques (macaca-mulatta). *Psychological Science,* **2,** 190–193.

Weddell, R. A., & Davidoff, J. B. (1991). A dyscalculic patient with selectively impaired processing of the numbers 7, 9, and 0. *Brain and Cognition,* **17,** 240–271.

Wynn, K. (1996). Infants' individuation and enumeration of actions. *Psychological Science,* **7,** 164–169.